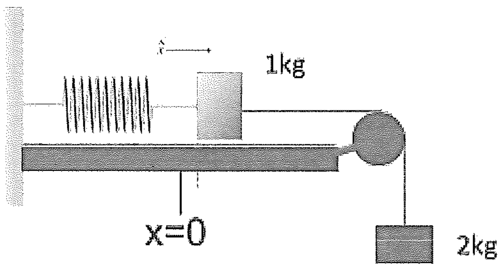
Physics 2310-01 Spring 2020 Exam 1

Name Solutions SHOW ALL WORK!

1) A 1kg mass is attached to a spring and rests on a frictionless surface. When the mass is connected to a massless rope (which passes over a massless, frictionless pulley and is finally attached to a 2kg mass), the spring extends by 1cm. The string is then cut. What is the period of small oscillations of the 1kg mass?



$$F_{sp} = -kx$$

$$-(2kg)g = -k(0.01\,m)$$

$$k = 1960\,\frac{N}{m}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1960\,\frac{N}{m}}{1kg}} = 44.27\,s^{-1}$$

$$\frac{1}{T} = f = \frac{\omega}{2\pi} \Rightarrow T = \frac{2\pi}{\omega} = 0.14\,s$$

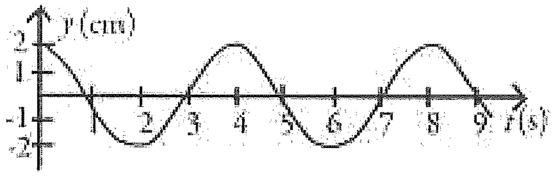
2) What is the velocity of the 1kg mass when it first passes x=0.0cm?

When it passes $x=0$, it will be moving to the left, and have as much kinetic energy as was stored as potential energy initially.

$$\therefore \tfrac{1}{2}mv^2 = \tfrac{1}{2}kA^2$$

$$v = \sqrt{\frac{k}{m}}\,A = (44.27\,s^{-1})(0.01\,m) = .44\,\frac{m}{s}$$

3) A wave is described by the equation: $y(x,t) = A\cos(kx - \omega t + \phi)$. Given that the velocity of the wave is ~~4~~ 1.0 m/s and $y(x = 2m, t)$ is plotted below, make a sketch of this wave at time t=1 s (be sure to scale the axes!).



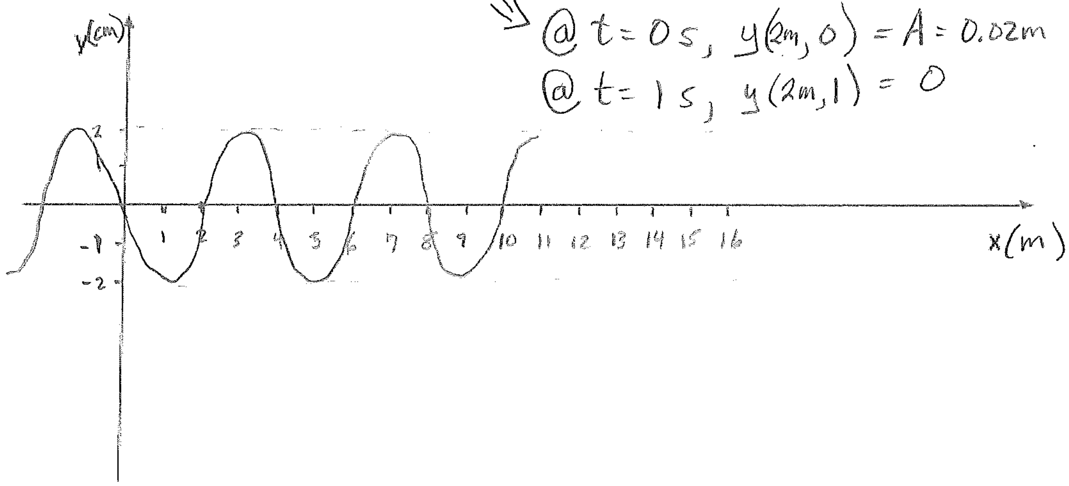
wave is moving to right

$\Rightarrow T = 4s \therefore f = \frac{1}{4} s^{-1}$

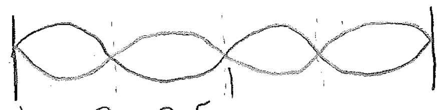
$v = \lambda f \Rightarrow \lambda = \frac{v}{f} = \frac{1 \, m/s}{\frac{1}{4} s^{-1}} = 4 \, m$

$\Rightarrow$ @ $t = 0s$, $y(2m, 0) = A = 0.02m$

@ $t = 1s$, $y(2m, 1) = 0$



4) A 1m long string with fixed ends is oscillating in its fourth harmonic (third overtone). Waves on these strings propagate at 75 m/s. Make a sketch of the vibrating string. What is the wavelength of the **sound waves** that this vibrating string causes?



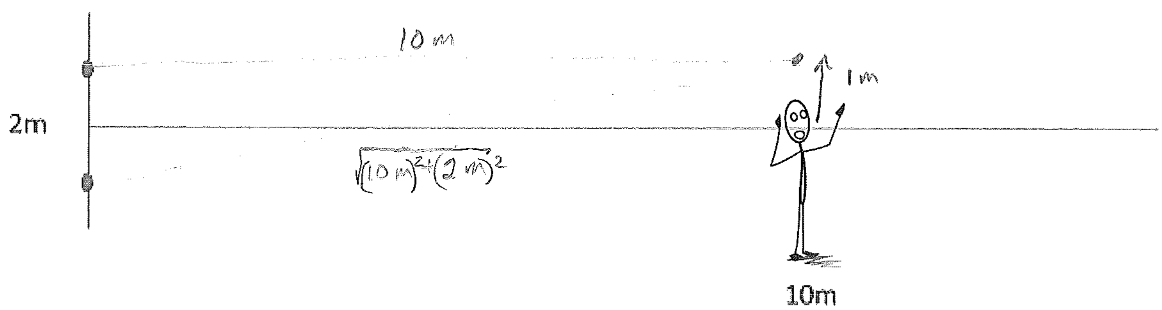
$\lambda = 0.5m$

$$V = 75\,\frac{m}{s} = \lambda f \Rightarrow f = \frac{75\,m/s}{0.5\,m} = 150\,Hz$$

$$V_{sound} = 344\,\frac{m}{s} = \lambda_{sound} \cdot f \Rightarrow \lambda_{sound} = \frac{344\,m/s}{150\,s^{-1}}$$

$$\lambda_{sound} = 2.29\,m$$

5) Two in-phase loudspeakers that emit sound with the same frequency are placed along a wall and are separated by a distance of 2.00 m. A person is standing 10.0 m away from the wall, equidistant from the loudspeakers. When the person moves 1.00 m parallel to the wall, she experiences destructive interference for the first time. What is the frequency of the sound?

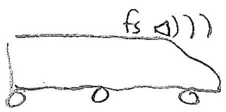


$$k(x_1 - x_2) = \frac{2\pi}{\lambda}(0.198m) = \pi \Rightarrow$$

$$\lambda = 2 \times 0.198\,m = 0.396\,m$$

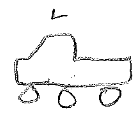
$$V = \lambda f \Rightarrow f = \frac{344\,m/s}{0.396\,m} = 868\,s^{-1}$$

6) A train engineer blows her horn as she approaches a truck stalled on the tracks. The horn emits an 800.00 Hz sound. She hears the horn reflected back from the truck at a frequency 810.00 Hz. What is the speed of the train?



$S \xleftarrow{+} L$ ①

$L \xrightarrow{+} S$ ②



$$① \quad f_L = \frac{V_w + V_L}{V_w + V_s} f_s = \frac{344\,m/s}{344\,m/s - V_t} f_s \quad \Big\}\ \text{What truck hears + reflects}$$

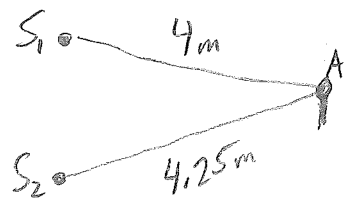
$$② \quad f_t = \frac{V_w + V_t}{V_w} f_L = \frac{V_w + V_t}{\cancel{V_w}} \cdot \frac{\cancel{V_w}}{V_w - V_t} f_s$$

$$(810\,Hz)(V_w - V_t) = (800\,Hz)(V_w + V_t)$$

$$10\,Hz\ V_w = 1610\,Hz \cdot V_t \Rightarrow V_t = \frac{10\,Hz}{1610\,Hz} \cdot 344\,m/s = 2.14\,m/s$$

7) A microphone is placed 4m from one speaker and 4.25m from a second speaker. Both speakers are emitting a 344Hz sound wave, but doing so completely out of phase. Draw a phasor diagram for the sound that the microphone hears.

$$\lambda = \frac{344\,m/s}{344\,s^{-1}} = 1m$$



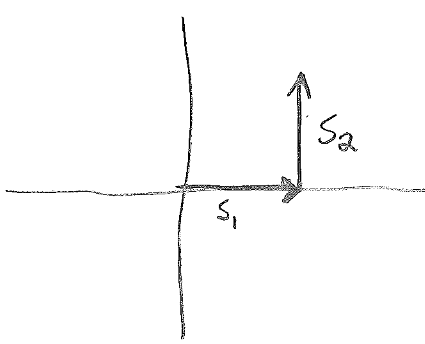
$$y_1(A,t) = A\sin(k(4m) - \omega t)$$

$$y_2(A,t) = A\sin(k(4.25m) - \omega t + \pi)$$

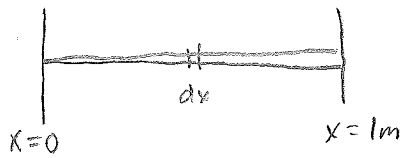
$$\delta\phi = \left(\frac{2\pi}{1m} \cdot 4m - \omega t\right) - \left(\frac{2\pi}{1m} \cdot 4.25m - \omega t + \pi\right)$$

$$= (8\pi - \omega t) - (8.5\pi - \omega t + \pi)$$

$$= \frac{1}{2}\pi$$



8) A non-uniform massive rope of length 1.0 m, has a mass density, μ that varies with position as $\mu = \frac{0.01kg}{m^2}x$. The rope is stretched between two walls with a tension of 100N. How much time does it take a pulse to travel from one end of the rope to the other? Ignore the effects of gravity.



The time for the pulse to pass dx is

$$dt = \frac{dx}{v}$$

where $V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{100N}{.01\frac{kg}{m^2}x}} = \sqrt{\left(10{,}000\frac{m^3}{s^2}\right)}\, x^{-1/2}$

$$\therefore dt = \frac{1}{100\frac{m^{3/2}}{s}} x^{1/2} dx$$

$$t = \frac{1}{100\frac{m^{3/2}}{s}} \int_0^{L=1m} x^{1/2} dx$$

$$= 0.01 \frac{s}{m^{3/2}} \frac{2}{3} \left[x^{3/2} \right]_0^{1m}$$

$$= \frac{0.02}{3} s$$

$$= 6.7 \text{ ms}$$