Scalar Product:

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta_{AB} = A_x B_x + A_y B_y + A_z B_z$$

Vector Product:

$$\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta_{AB}$$
$$= \left(A_y B_z - A_z B_y\right)\hat{i}$$
$$+\left(A_z B_x - A_x B_z\right)\hat{j}$$
$$+\left(A_x B_y - A_y B_x\right)\hat{k}$$

Equations of motion:

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0{}^2 + 2a(x - x_0)$$

Radial Acceleration:

$$a_{rad} = \frac{v^2}{r}$$

Newton’s second law

$$\sum \vec{F} = m\vec{a}$$

Magnitude of kinetic friction

$$F_{f_k} = \mu_k F_N$$

Magnitude of static friction

$$F_{f_s} \le \mu_s F_N$$

Definition of work

$$W = \int \vec{F}\cdot d\vec{x}$$

Definition of kinetic energy:

$$KE = \frac{1}{2}mv^2$$

Change in gravitational potential energy:

$$\Delta U_g = mg\Delta y$$

Elastic potential energy:

$$U_{el} = \frac{1}{2}kx^2$$

Work-Energy Theorem:

$$W = \Delta U + \Delta KE$$

Center-of-mass position

$$X_{COM} = \frac{1}{M}\sum_{i=1}^{n} x_i m_i$$

Definition of momentum

$$\vec{p} = m\vec{v}$$

Conservation of momentum

$$\vec{p}_i = \vec{p}_f$$

Definition of torque

$$\vec{\tau} = \vec{r}\times\vec{F}$$

Newton’s second law for rotation

$$\sum \vec{\tau} = I\vec{\alpha}$$

Conditions for rolling:

$a_{COM} = \alpha R$ and $v_{COM} = \omega R$

Angular momentum:

$\vec{L} = \vec{r}\times\vec{p}$ or $\vec{L} = I\vec{\omega},$ where $I = \sum_i m_i r_i^2$

Newton’s Law of Gravitation:

$F_G = \frac{Gm_1m_2}{r^2}$ and $U_G = -\frac{Gm_1m_2}{r}$ with $U_G = 0$ at infinity

Bernoulli's Equation:

$$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

Equation for Simple Harmonic Motion:

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

Solution for above equation:

$$x(t) = A\cos(\omega t + \varphi)$$

Where,

$$\omega = 2\pi f = \frac{2\pi}{T}$$

For a spring mass oscillator,

$$\omega = \sqrt{\frac{k}{m}}$$

For a simple pendulum,

$$\omega = \sqrt{\frac{g}{L}}$$

Wave Equation:

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

Solution to above equation:

$$y(x,t) = A\cos(kx - \omega t)$$

Where,

$$k = \frac{2\pi}{\lambda},\quad \omega = 2\pi f,\quad v = \lambda f$$

Standing waves on fixed string:

$$y(x,t) = A_{SW}\sin(kx)\sin(\omega t)$$

$$f_n = n\frac{v}{2L}$$

Doppler Effect:

$$f_L = \frac{v+v_L}{v+v_S} f_S$$

$$T\left({}^oC\right) = \frac{5}{9}\left(T\left({}^oF\right) - 32\right)$$

$$T\left(K\right) = T\left({}^oC\right) + 273.15$$

$$\Delta L = \alpha L_0 \Delta T$$

$$\Delta V = \beta V_0 \Delta T$$

$$Q = mc\Delta T = nC\Delta T$$

$$Q_{F/V} = \pm mL_{F/V}$$

$$H = \frac{dQ}{dt} = k\frac{A}{L}\left(T_H - T_C\right)$$

$$pV = nRT$$

$$K_{tr} = \frac{3}{2}nRT$$

$C_V = \frac{3}{2}R$ ideal monatomic gas

$C_V = \frac{5}{2}R$ ideal diatomic gas w/o vibration

$$W = \int_{V_1}^{V_2} p\,dV$$

$$\Delta U = Q - W$$

$\Delta U = nC_V\Delta T$ for ideal gas

$\left.\begin{array}{l} pV^{\gamma} = const \\ TV^{\gamma-1} = const \end{array}\right\}$ adiabatic process

$$e = \frac{W}{Q_H} = 1 - \left|\frac{Q_C}{Q_H}\right|$$

$$e_{Carnot} = 1 - \left|\frac{T_C}{T_H}\right|$$

$$\Delta S = \int_1^2 \frac{dQ}{T}$$

$$S = k\ln w$$

$$R = 8.314\, J/mol\cdot K$$

$$N_A = 6.02\times 10^{23}\, molecules/mole$$

1 atm = 101 325 N / (m^2) = $1.01\text{x}10^5$ Pa

$1/4\pi\varepsilon_0 = 8.99 \text{ x } 10^9$ Nm^2/C^2

e = $-1.602 \text{ x } 10^{-19}$ C

$$\vec{F}_E = q\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}\hat{r}$$

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$\Delta U = q\Delta V$$

$$\vec{E} = -\left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right)$$

$$Q = CV$$

$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$ series

$C_{eq} = C_1 + C_2 + C_3 + \ldots$ parallel

$$U = \frac{1}{2}CV^2$$

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

$$E = \frac{E_0}{K}$$

$$I = \frac{dq}{dt}$$

$$\vec{J} = nq\vec{v}_d$$

$$\rho = \frac{E}{J}$$

$$R = \frac{\rho L}{A}$$

$$V = IR$$

$$P = VI$$

$R_{eq} = R_1 + R_2 + R_3 + \ldots$ series

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$ parallel

$q = C\mathcal{E}\left(1 - e^{-t/RC}\right)$ charging

$q = Q_0 e^{-t/RC}$ discharging

$$\vec{F} = q\vec{v}\times\vec{B}$$

$$\Phi_B = \int \vec{B}\cdot d\vec{A}$$

$$d\vec{F} = Id\vec{l}\times\vec{B}$$

$$\vec{\tau} = \vec{\mu}\times\vec{B}, \quad \vec{\mu} = NI\vec{A}$$

$$U = -\vec{\mu} \cdot \vec{B}$$

$$V_s = V_p \frac{N_s}{N_p}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \hat{r}}{r^2}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + i_D \right)$$

$$\oint \vec{\mathrm{E}} \cdot d\vec{l} = \mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$i_D = \varepsilon \frac{d\Phi_E}{dt}$$

$$\mathcal{E}_2 = -M \frac{di_1}{dt} \text{ and } \mathcal{E}_1 = -M \frac{di_2}{dt}$$

$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_1 \Phi_{B1}}{i_2}$$

$$\mathcal{E} = -L \frac{di}{dt},$$

$$L = \frac{N\Phi_B}{i}$$

$$U = \frac{1}{2} LI^2, \qquad u_E = \frac{1}{2\mu_0} B^2$$

$$\frac{di}{dt} = \frac{\mathcal{E}}{L} e^{-Rt/L}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$I_{RMS} = \frac{1}{\sqrt{2}} I \quad \text{for } i = I \cos\left(\omega t\right)$$

$$V_{RMS} = \frac{1}{\sqrt{2}} V \quad \text{for } v = V \cos\left(\omega t\right)$$

$$V_R = IR$$

$$V_L = IX_L, \quad \text{where } X_L = \omega L$$

$$V_C = IX_C, \quad \text{where } X_C = \frac{1}{\omega C}$$

$$V = IZ, \quad \text{where } Z = \sqrt{R^2 + \left(X_L - X_C \right)^2}$$

$$P_{\text{Avg}} = \frac{1}{2} VI \cos\varphi, \quad \tan\varphi = \frac{X_L - X_C}{R}$$

Calculus

Derivatives:

$$\frac{d}{dx}x^n = nx^{n-1} \qquad \frac{d}{dx}\ln ax = \frac{1}{x} \qquad \frac{d}{dx}e^{ax} = ae^{ax}$$

$$\frac{d}{dx}\sin ax = a\cos ax \qquad \frac{d}{dx}\cos ax = -a\sin ax$$

Integrals:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1) \qquad \int \frac{dx}{x} = \ln x \qquad \int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax \qquad \int \cos ax\,dx = \frac{1}{a}\sin ax \qquad \int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a}$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right) \qquad \int \frac{dx}{x^2 + a^2} = \frac{1}{a}\arctan\frac{x}{a} \qquad \int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{1}{a^2}\frac{x}{\sqrt{x^2 + a^2}}$$

$$\int \frac{x\,dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}$$

Physics 161-001 Spring 2014 Exam 3

Name: ___________________________________ Box#_____________

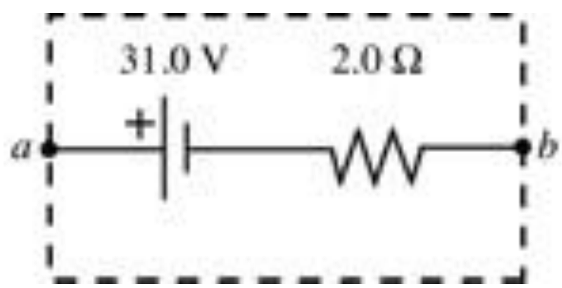
Multiple Choice (5 points each):

1) A tube of mercury with resistivity 7.84 × 10^{-6} Ωm has an electric field inside the column of mercury of magnitude 8 V/m that is directed along the length of the tube. How much current is flowing through this tube if its radius is 6.0 mm?

A) 4.80 A
B) 6.00 A
C) 10.0 A
D) 12.3 A
E) 19.2 A
F) 25.0 A
G) 55.4 A
H) 87.3 A
I) 115 A
J) 134 A

$$J = \frac{E}{\rho} = \frac{8V/m}{7.84\times10^{-6}\Omega m} = 1.0\times10^{6}\, A/m^{2} \text{ and}$$
$$I = J\cdot A = \left(1.0\times10^{6}\, A/m^{2}\right)\left(\pi\left(6\times10^{-3}m\right)^{2}\right) = 115A.$$

2) The emf and the internal resistance of a battery are as shown in the figure. When the terminal voltage Vab is equal to 21.0 V, what is the current through the battery?



A) 1.2 A
B) 4.3 A
C) 5.0 A
D) 10.0 A
E) 15.5 A
F) 19.1 A
G) 23.2 A
H) 26.0 A
I) 52.0 A
J) 104 A

$$V_{ab} = 21.0V = \mathcal{E} - Ir = 31V - I\left(2\Omega\right) \Rightarrow$$
$$I = 5.0A$$

3) A proton moving in the positive x direction enters a magnetic field. If the proton experiences a magnetic deflection in the negative y direction, the magnetic field in this region is

A) in the direction of the +x axis.
B) in the direction of the -x axis.
C) in the direction of the +y axis.
D) in the direction of the -y axis.
E) in the direction of the +z axis.
F) in the direction of the -z axis.
G) in any direction perpendicular to the proton velocity.
H) zero.
I) undefined.

$F_B = q\vec{v} \times \vec{B}$. Since the proton is positively charged, the direction is in the same direction given by the right hand rule.

4) If the current density in a wire of radius R is given by $J = J_0 + kr$, $0 < r < R$, what is the total current in the wire?

A) $kR^2/2$
B) kR^2
C) $J_0R + kR^2/2$
D) $J_0R^2 + kR^3/3$
E) $J_0\pi R^2 + k2\pi R^3/3$
F) $J_0 2\pi R + k2\pi R^2/2$
G) $J_0\pi R + k2R^2/2$
H) $k2\pi R^3/3$
I) $kR^3/3$
J) $J_0(k2\pi R^3/3)$

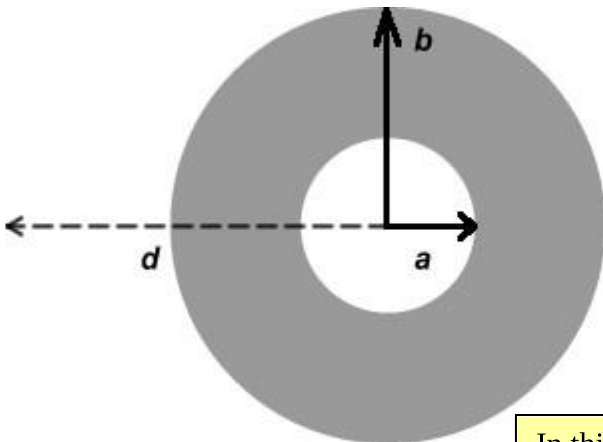
$$I = \int_0^R J(r)\,dA = \int_0^R J(r)\,2\pi r dr$$

$$= \int_0^R (J_0 + kr)\,2\pi r dr = \int_0^R J_0 2\pi r dr + \int_0^R kr 2\pi r dr$$

$$= J_0 2\pi \int_0^R r dr + 2\pi k \int_0^R r^2 dr$$

$$= J_0 \pi R^2 + 2\pi k\, R^3/3$$

5) The figure shows the cross-section of a hollow cylinder of inner radius a = 1.0 cm and outer radius b = 2.0 cm. A uniform current density of 1.0 A/ cm^2 flows through the cylinder parallel to its axis. Calculate the magnitude of the magnetic field at a distance of d = 1.0 m from the axis of the cylinder. ($\mu_0 = 4\pi \times 10^{-7}$ T · m/A)



A) 1.1 x 10^{-5} T
B) 1.5 x 10^{-5} T
C) 1.1 x 10^{-6} T
D) 1.9 x 10^{-6} T
E) 6.3 x 10^{-6} T
F) 9.7 x 10^{-6} T
G) 2.2 x 10^{-7} T
H) 4.8 x 10^{-7} T
I) 9.1 x 10^{-7} T
J) 0 T

In this case, J is uniform, so,

$$I = JA = J\left(\pi b^2 - \pi a^2\right) = \left(1A/cm^2\right)\pi\left(b^2 - a^2\right)$$

$$= \left(1A/cm^2\right)\pi\left(4cm^2 - 1cm^2\right) = 9.4A$$

Then, from symmetry, we use Ampere's law to find that:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

$$B2\pi r = \left(4\pi \times 10^{-7}\, T \cdot m/A\right) \times (9.4A) \Rightarrow$$

$$B = 1.9 \times 10^{-6} T$$

6) Calculate the current through a 1.0-m long 22 gauge (having radius 0.321 mm) nichrome wire if it is connected to a 3.0-V battery. The resistivity of nichrome is $100 \times 10^{-8}\ \Omega \cdot$ m.

A) 1 A
B) 2 A
C) 3 A
D) 4 A
E) 5 A
F) 6 A
G) 7 A
H) 8 A
I) 9 A
J) 10 A

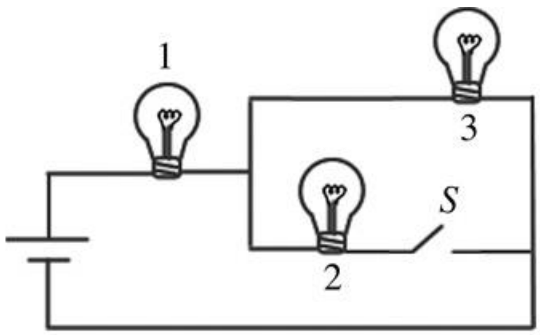
$$R = \frac{\rho L}{A} = \frac{100 \times 10^{-8}\Omega m \cdot 1m}{\pi\left(3.21 \times 10^{-4} m\right)^2} = 3.1\Omega$$

and,

$$V = IR \Rightarrow$$

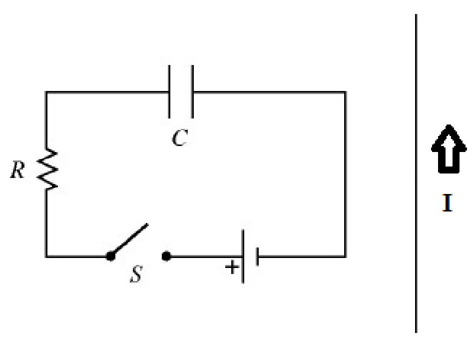
$$I = \frac{V}{R} = \frac{3V}{3.1\Omega} = 1A$$

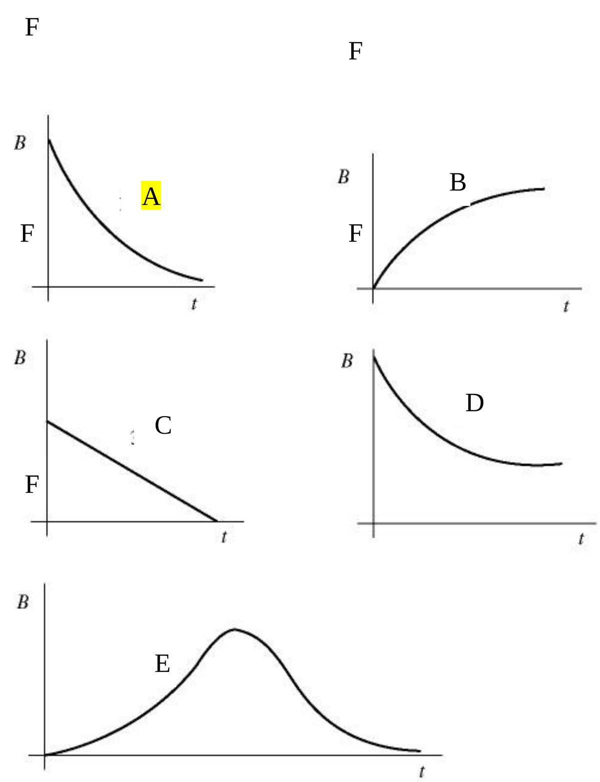
7) The figure shows three identical light bulbs connected to a battery having a constant voltage across its terminals. What happens to the brightness of light bulb 3 when the switch *S* is closed?



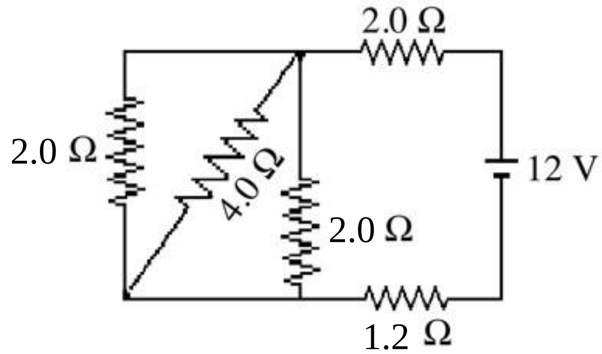
A) Momentarily goes up then back to its original brightness.
B) Momentarily goes down then back to its original brightness.
C) Permanently gets brighter.
D) Permanently gets dimmer.
E) No change.

8) A current is running through a wire next to the circuit shown in the figure with the switch *S* open and the capacitor uncharged. The battery has no appreciable internal resistance. Which one of the following graphs best describes the magnitude of the force on the wire as a function of time *t* after closing the switch?





9) For the circuit shown in the figure, determine the current in the 4.0-Ω resistor.



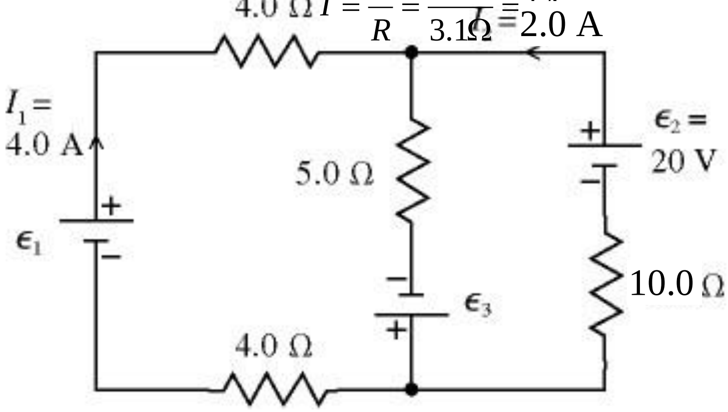
A) 0.1 A
B) 0.2 A
C) 0.4 A
D) 0.6 A
E) 0.8 A
F) 1.0 A
G) 1.2 A
H) 1.4 A
I) 1.6 A
J) 1.8 A

We first find the equivalent resistance of the three resistors in parallel: $\frac{1}{R_{eq}} = \frac{1}{2\Omega} + \frac{1}{2\Omega} + \frac{1}{2\Omega} \Rightarrow R_{eq} = \frac{4}{5}\Omega$, then the equivalent resistance of the three in series: $R_{eq} = \frac{4}{5}\Omega + 2\Omega + 1.2\Omega = 4\Omega$

and, then the current through the entire circuit: I = V/R = 3A. Then, the voltage across the resistors in parallel is 12/5V and so the current through the 4.0-Ω resistor is 3/5 A.

10) Consider the circuit shown in the figure. Note that two currents are shown. Calculate the emf $V = 3IR \Rightarrow$

$I = \frac{V}{R} = \frac{3V}{3.1\Omega} =$

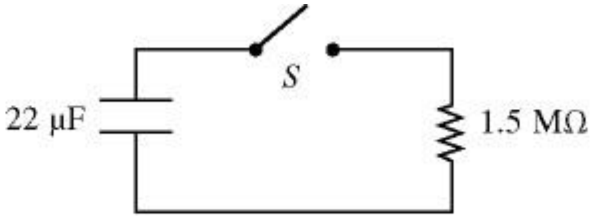


A) 50 V
B) 48 V
C) 44 V
D) 42 V
E) 40 V
F) 38 V
G) 36 V
H) 34 V
I) 32 V
J) 30 V

$V_{ab} = 21.0V = \mathcal{E} - Ir = 31V - I(2\Omega) \Rightarrow$

$I = 5.0A$

11) For the circuit shown in the figure, the switch *S* is initially open and the capacitor voltage is 80 V. The switch is then closed at time $t = 0$. What is the charge on the capacitor when the current in the circuit is 13 μA?



A) 110 μC
B) 140 μC
C) 200 μC
D) 220 μC
E) 280 μC
F) 330 μC
G) 390 μC
H) 430 μC
I) 470 μC
J) 500 μC

$$q = Q_0 e^{-t/RC} \text{ and } i = \frac{dq}{dt} = -\frac{Q_0}{RC} e^{-t/RC}.$$

Now, at some time, i = 13μA, or:

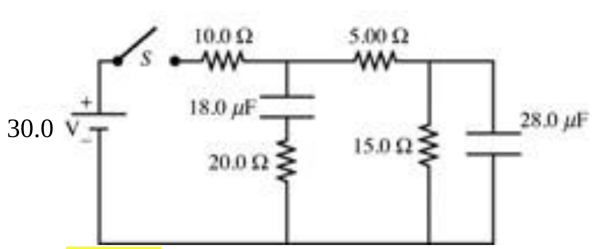
$$i = \frac{Q_0}{RC} e^{-t/RC} = 13\mu A \Rightarrow$$

$$e^{-t/RC} = \frac{RC}{Q_0} 13\mu A = \frac{RC}{CV_0} 13\mu A = \frac{R}{V_0} 13\mu A$$

then,

$$q = Q_0 e^{-t/RC} = Q_0 \frac{R}{V_0} 13\mu A = C \cdot R \cdot 13\mu A = 430\mu C$$

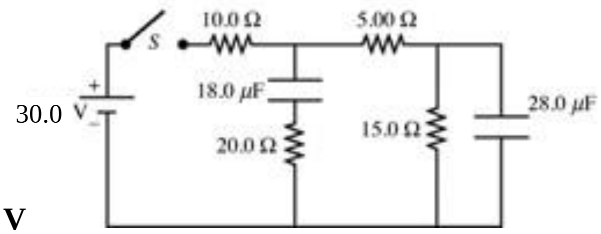
12) For the circuit shown in the figure, the capacitors are all initially uncharged, the connecting leads have no resistance, the battery has no appreciable internal resistance, and the switch *S* is originally open. Just after closing the switch *S*, what is the current in the 15.0-Ω resistor?



A) 0 A
B) 0.2 A
C) 0.4 A
D) 0.6 A
E) 0.8 A
F) 1.0 A
G) 1.2 A
H) 1.4A
I) 1.6 A
J) 1.8 A

Immediately after the switch is closed, the voltage drop across the capacitor is 0V, so no current through the 15 Ω resistor.

13) After the switch S has been closed for a very long time, what is the potential difference across the 28.0-μF capacitor?



A) 0.0 V
B) 4.3 V
C) 5.0 V
D) 10.0 V
E) 15.0 V
F) 19.1 V
G) 23.2 V
H) 26.0 V
I) 30.0 V
J) 104 V

For long times, no current passes through the capacitors, so the current just goes through the three resistors in series. The equivalent resistance is just 30 Ω, with 30V means 1 A is passing through each resistor. Then the voltage drop across the 15 Ω resistor is just 15 V.

14) A charge is accelerated from rest through a potential difference V and then enters a uniform magnetic field oriented perpendicular to its path. The field deflects the particle into a circular arc of radius R. If the accelerating potential is doubled to $2V$, what will be the radius of the circular arc?

A) 2R
B) R/2
C) 4R
D) R/4
E) $\sqrt{2}$ R
F) R / $\sqrt{2}$
G) R
H) 3R
I) $\sqrt{3}$ R
J) R/ $\sqrt{3}$

$$KE = \frac{1}{2}mv^2 = qV \Rightarrow$$

and

$$v = \sqrt{\frac{2qv}{m}}$$

$$\vec{F}_B = q\vec{v} \times \vec{B} \Rightarrow$$

$$= qvB = ma = m\frac{v^2}{R} \Rightarrow$$

$$R = \frac{mv^2}{qvB} = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

15) A circular coil of wire of 100 turns and diameter 10.0 cm carries a current of 1.0 A. It is placed in a magnetic field of 1 T with the plane of the coil making an angle of 45° with the magnetic field. What is the magnetic torque on the coil?

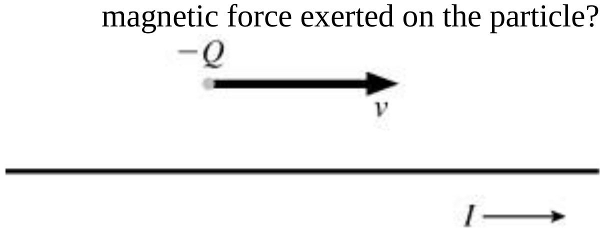
A) 2.0 Nm
B) 2.2 Nm
C) 2.4 Nm
D) 2.6 Nm
E) 2.8 Nm
F) 3.0 Nm
G) 3.2 Nm
H) 3.4 Nm
I) 3.6 Nm
J) 3.8 Nm

$\vec{\tau} = \vec{\mu} \times \vec{B}$ and

$\mu = NIA = 100 \cdot 1.0A \cdot \pi (0.1m)^2 = 3.14Am^2$ so

$|\vec{\tau}| = \mu B \sin\theta = 3.14Am^2 \cdot 1T \cdot \sin 45 = 2.22Nm$

16) A negatively charged particle is moving to the right, directly above a wire having a current flowing to the right, as shown in the figure. In which direction is the magnetic force exerted on the particle?



A) up
B) down
C) out of the page
D) into the page
E) no magnetic force

The B-field from the wire is coming out of the page at the charge, and from $\vec{F} = q\vec{v} \times \vec{B}$, with q negative, then the force is upwards.

17) A *large* number of *very long* wires of diameter 1mm are laid side-by-side to form a plane. If 10.5 A of current is passed through each wire (in the same direction), what is the magnitude of the magnetic field 10cm above (and in the middle of) the plane? ($\mu_0 = 4\pi \times 10^{-7}$ T · m/A)

A) 1.6×10^{-4} T
B) 5.2×10^{-4} T
C) 7.9×10^{-4} T
D) 2.6×10^{-3} T
E) 4.7×10^{-3} T
F) 6.6×10^{-3} T
G) 9.2×10^{-3} T
H) 2.1×10^{-2} T
I) 4.4×10^{-2} T
J) 6.5×10^{-2} T

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

$$2BL = \left(4\pi \times 10^{-7}\, T \cdot m/A\right) \times \frac{(10.5A)}{1 \times 10^{-3} m} L \Rightarrow$$

$$B = \left(2\pi \times 10^{-7}\, T \cdot m/A\right) \times \frac{(10.5A)}{1 \times 10^{-3} m}$$

$$= 6.6 \times 10^{-3} T$$