Solutions for Homework 14

1)

$V = IR = I_0 R \cos\omega t$

$\therefore \mathcal{E} = I_0 R$

Since $\mathcal{E} = 170$ volts, it follows that $I_0 = \frac{170\,V}{10\,\Omega} = \boxed{17\,A}$

It follows also that $\boxed{V(t) = 170 \text{ Volts} \cos(\omega t)}$

Power $= V(t) I(t) = 170 \cos(\omega t) \cdot 17 \cos(\omega t)$

$= 2.89 \times 10^3 \cos^2(\omega t)$ watts

($\cos^2$ averages to $\frac{1}{2}$.)

$\overline{P} = \frac{1}{2}(2.89 \times 10^3 \text{ watts}) = \boxed{1.45\,kW}$

2) $X_L = \omega L = 377 \frac{\text{rad}}{\text{s}} \cdot 1\,H = 377$ Ohms

$V = \underbrace{I_0 X_L}_{170 \text{ volts}} \cos(\omega t + \pi/2)$ ← $\mathcal{E} = LI$; 90°

$\therefore I_0 = \frac{170\,V}{377\,\Omega} = \boxed{0.45\,A}$

$\boxed{V(t) = 170 \text{ volts} \cos(\omega t + \pi/2)}$

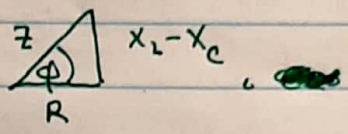
$\boxed{\overline{P} = 170 \text{ volts} \cdot 0.45\,A \cdot \overline{\cos(\omega t + \pi/2)\cos(\omega t)} = 0}$

3)

$V(t) = \mathcal{E} \cos(\omega t + \phi)$ where $\mathcal{E} = 170$ Volts, but what is ϕ?

$$V - I_0 R \cos(\omega t) - I_0 X_L \cos(\omega t + 90^\circ) - I_0 X_C \cos(\omega t - 90^\circ) = 0$$

$$V = I_0 Z \cos(\omega t + \phi)$$

where 

$X_L = 377\,\Omega$

$X_C = \dfrac{1}{\left(377 \frac{rad}{s}\right)(1\times10^{-5})} = 265\,\Omega$

$R = 10\,\Omega$

$Z = \sqrt{(10)^2 + (377-265)^2} = \boxed{112\,\Omega}$

$I_0 (112\,\Omega) = \mathcal{E} = 170$ Volts

& $I_0 = \dfrac{170}{112} = \boxed{1.51\ A}$

$\phi = \tan^{-1}\left(\dfrac{377-265}{10}\right) = 85^\circ$

$$\boxed{V = 170 \text{ volts} \cos(\omega t + 85^\circ)}$$

$\overline{P} = \overline{I(t)\,V(t)} = (170 \text{ Volts})(1.51 A) \overline{\cos(\omega t + 85^\circ)\cos(\omega t)}$

$\overline{\cos(\omega t + 85^\circ)\cos(\omega t)} \rightarrow \dfrac{(\cos(\omega t)\cos(85^\circ) - \sin(\omega t)\sin(85^\circ))}{2}$

$= 257 \text{ watts} \cdot \cos(85^\circ)\cos(\omega t)$

$= \frac{1}{2} \cdot 257 \text{ watts} \cos(85^\circ) = \boxed{11.2\,W}$

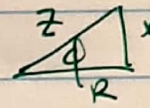
4)

$$\bar{P} = \overline{\mathcal{E}\cos(\omega t+\phi) I_0 \cos(\omega t)}$$

$$= \mathcal{E} I_0 \cos\phi \; \overline{\cos^2(\omega t)}$$

$$= \frac{1}{2}\mathcal{E} I_0 \cos\phi$$

$$= \frac{1}{2}\mathcal{E}\cdot\frac{\mathcal{E}}{Z}\cos\phi \leftarrow$$



$\cos\phi = R/Z$

$$= \frac{1}{2}\frac{\mathcal{E}^2}{Z}\cdot\frac{R}{Z}$$

$$= \frac{1}{2}\frac{(170)^2\, 10}{\left(\sqrt{(10)^2 + \left(\omega\cdot 1 - \frac{1}{\omega\cdot(10^{-5})}\right)^2}\right)^2}$$

$$\bar{P} = \frac{(170)^2\cdot 10/2}{\left[100 + \left(\omega - \frac{10^5}{\omega}\right)^2\right]} \leftarrow 1.44\times 10^5$$

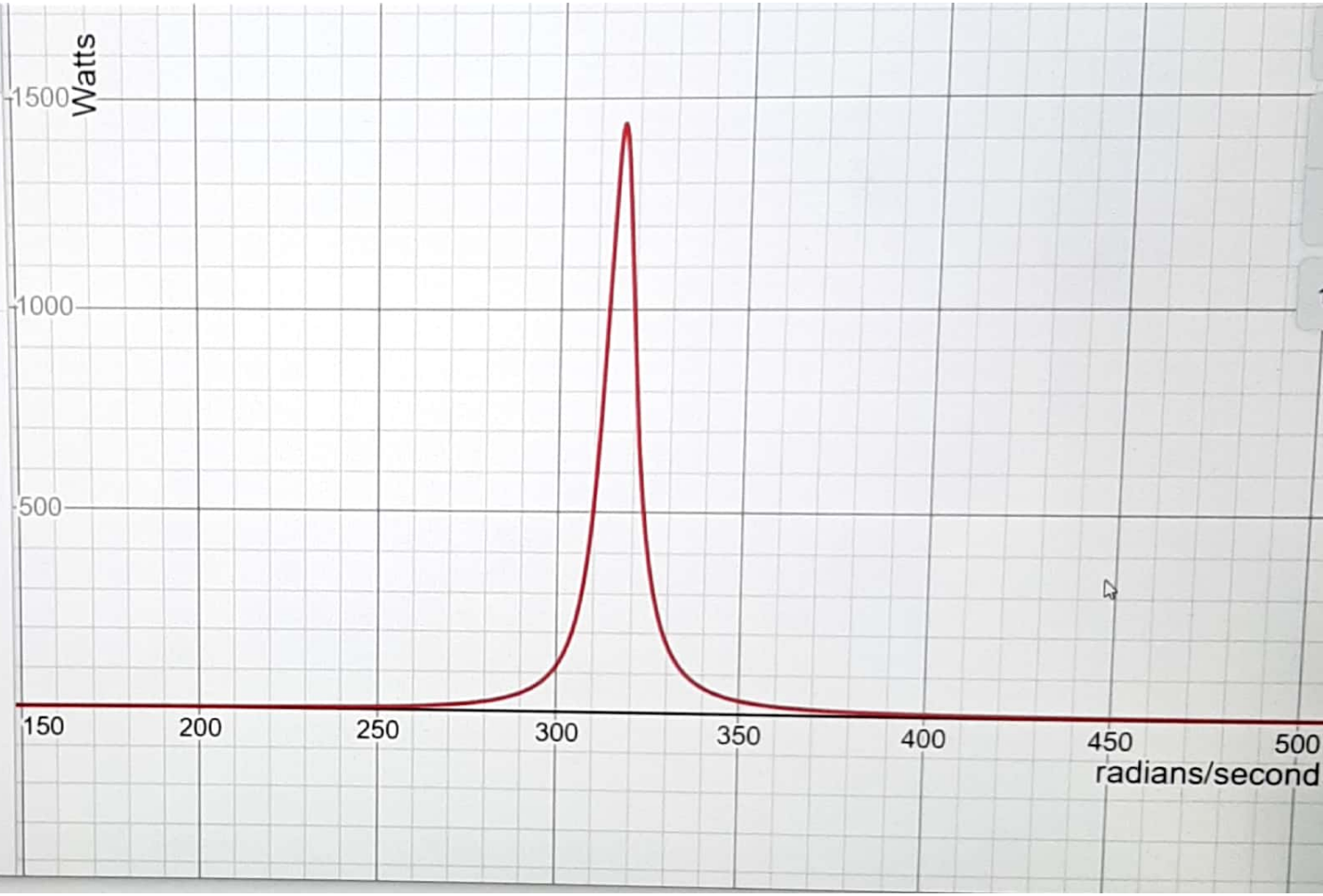
↑ radians/s.

see graph from DESMOS

Resonance at $\omega^2 = 10^5$

$\omega = 316$ rad/s

$$1.44 \cdot \frac{10^5}{\left(100 + \left(x - \frac{10^5}{x}\right)^2\right)}$$



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Solutions to problems 5 and 6

5. Below are the four Maxwell equations.

$$(i)\ \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

$$(ii)\ \oint \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

$$(iii)\ \oint \vec{B} \cdot d\vec{A} = 0$$

$$(iv)\ \oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enclosed}} + \mu_0 \epsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$

What are the names/people associated with these four laws?

(i) Gauss's Law, (ii) Faraday's law, (iii) Gauss's law for magnetism, (iv) Ampere's law (as modified by Maxwell).

Which of the two originated from Coulomb's law? (i) and (ii)
Which of the two originated from the Biot-Savart law? (iii) and (iv)
Which is known as Ampere's law? (iv)
Which is Gauss's law? (i)
Which is sometimes called "Gauss's law for magnetic fields?" (iii)
Which is Faraday's law? (ii)
Which of the four laws was modified by Maxwell on purely theoretical grounds? (iv).

5. Electromagnetic waves travel at a speed given by $\sqrt{\frac{1}{\mu_0 \epsilon_0}}$ in free space. Insert the values of μ_0 and ϵ_0 and calculate the speed in meters per second (and also in miles per second).

$\epsilon_0 = 1/(4\pi \times 9 \times 10^9)\ \text{C}^2/\text{Nm}^2$
$\mu_0 = 4\pi \times 10^{-7} \text{Ns}^2/\text{C}^2$
$\sqrt{\frac{1}{\epsilon_0 \mu_0}} = \sqrt{9 \times 10^{16} \text{m}^2/\text{s}^2} = 3 \times 10^8\ \text{m/s}$