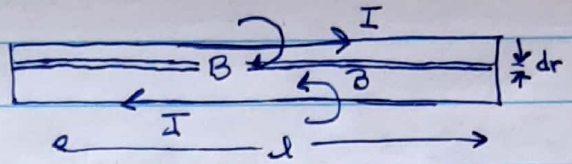
Solutions for Homework 13 — 5/1/2022

1) (a) (This calculation was worked out in class on Wed.)



$$\Phi = \int_{\text{loop area}} \vec{B}\cdot d\vec{A} = \int B_{\text{from top wire}}\, dA + \int B_{\text{from bottom wire}}\, dA$$

$$= 2\int B_{\text{from top wire}}\, dA$$

$$= 2\int_{r=a}^{w} \frac{\mu_0 I}{2\pi r}\cdot l\,dr$$

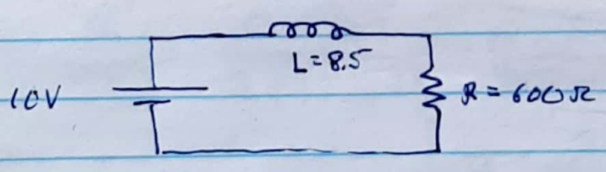
$$= \frac{\mu_0 I l}{\pi}\int_a^w \frac{dr}{r} = \frac{\mu_0 I l}{\pi}\ln(w/a)$$

$$L = \Phi/I = \boxed{\frac{\mu_0 l}{\pi}\ln(w/a)}$$

(b) $L = \dfrac{4\pi\times10^{-7}\cdot 300\times10^{3}}{\pi}\ln(5/0.01)$

$= \boxed{0.75\text{ H}}$

2)



$$V - L\frac{dI}{dt} - IR = 0$$

$$L\frac{dI}{dt} = V - IR$$

$$\int_0^I \frac{L\,dI/dt}{V-IR} = \int 1\cdot dt = t$$

$u = V - IR \quad du = -R\,dI$

$$-\frac{L}{R}\int_{u=V}^{u=V-IR}\frac{du}{u} = t$$

$$\ln\left(\frac{V-IR}{V}\right) = -\frac{R}{L}t$$

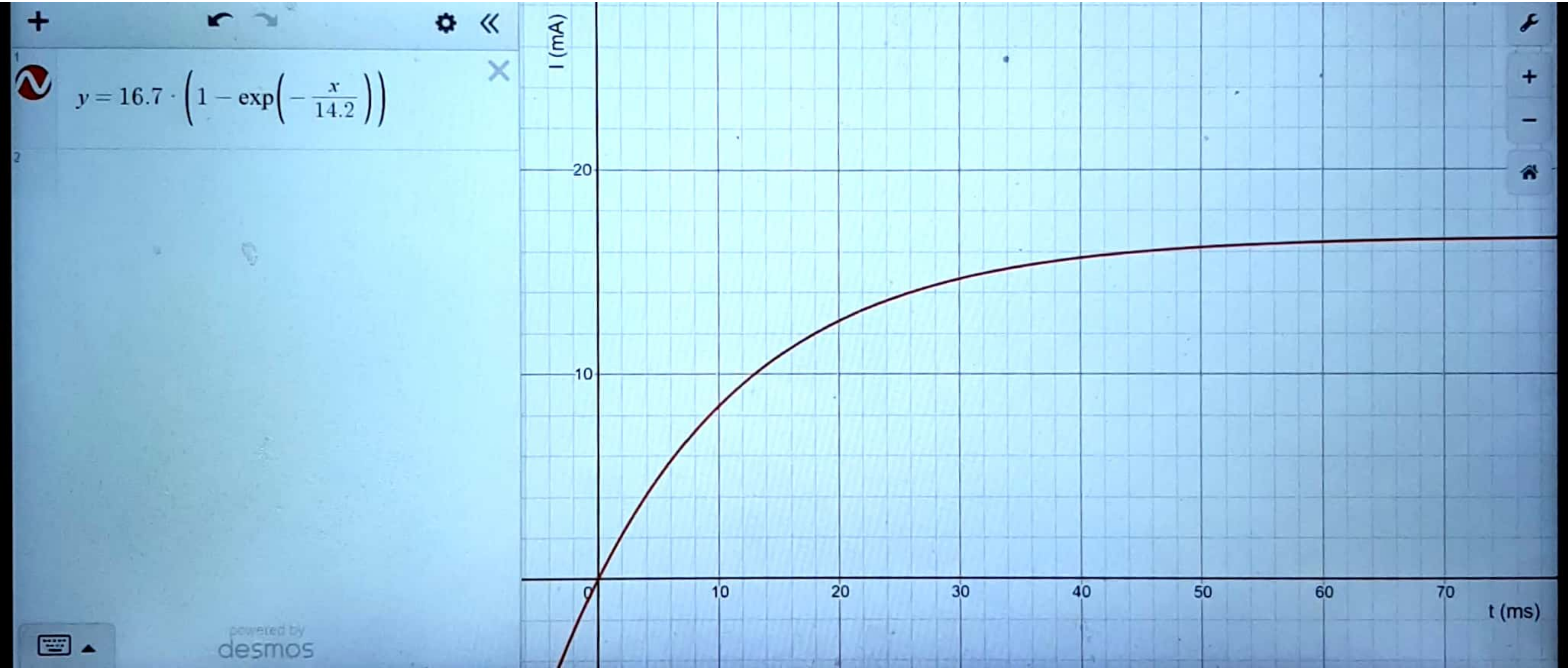
$$1 - I\frac{R}{V} = e^{-\frac{R}{L}t}$$

$$I = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$\boxed{16.7\text{mA}} = I(\infty)$

$\frac{R}{L} = 70.6\text{ s}^{-1}$

$\tau = L/R = \boxed{14.2\text{mS}}$.


y = 16.7 · (1 − exp(− x / 14.2))
I (mA)
20
10
0
10
20
30
40
50
60
70
t (ms)
powered by
desmos

3)

(a) $B_1 = \frac{\mu N_1 I_1}{l}$; $B_2 = \frac{\mu N_2 I_2}{l}$

(b) $\frac{V(t) - I_1 R_1}{N_1} = \frac{d}{dt}\left[\cancel{N_1}(B_1 + B_2)A\right]$

$-\frac{-I_2 R_2}{N_2} = \frac{d}{dt}\left[\cancel{N_2}(B_1 + B_2)A\right]$

$$\left(\frac{V(t) - I_1 R_1}{N_1}\right) - \left(\frac{-I_2 R_2}{N_2}\right) = 0$$

$$\boxed{V(t) - I_1 R_1 = -\frac{N_1}{N_2}(I_2 R_2)}$$

(c) $\frac{N_2}{N_1}(13{,}000) = 240$

$$\boxed{\frac{N_2}{N_1} = \frac{1}{54}}$$

4) $\frac{Q^2}{2C} = \frac{(5\times10^{-2}c)^2}{2(1\times10^{-3})} = \boxed{1.25\,J} = \frac{1}{2}LI^2$

$I = \sqrt{\frac{2(1.25J)}{1.01}} = \boxed{1.6\,A}$

5) (a) $-\frac{Q}{C} = L\frac{dI}{dt} = L\frac{d}{dt}(dQ/dt) = L\frac{d^2Q}{dt^2}$

$$\boxed{\frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0}$$

(b) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3}\cdot10^{-5}}} = \boxed{10^4\ rad/s.}$

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c) $Q = Q_0 \cos(\omega_0 t)$

$$I = \frac{dQ}{dt} = -Q_0 \omega_0 \sin \omega_0 t$$

$$\frac{dI}{dt} = -Q_0 \omega_0^2 \cos(\omega_0 t)$$

$$L\frac{dI}{dt} = -Q_0 L \omega_0^2 \cos(\omega_0 t)$$

$$= -Q_0 L \cdot \frac{1}{LC} \cos(\omega_0 t)$$

$$= -\boxed{\frac{Q_0}{C}} \cos(\omega_0 t)$$

$$\left| L\frac{dI}{dt} \right|_{max} = \frac{Q_0}{C} = \frac{1\times10^{-3}\,C}{1\times10^{-5}\,F} = \underline{100 \text{ volts}}$$

$$\left| \frac{Q}{C} \right|_{max} = \frac{Q_0}{C} = \underline{100 \text{ volts}}$$

$$+\frac{Q}{C} = -L\frac{dI}{dt}$$

$$\Delta V_{capacitor} + \Delta V_{inductor} = 0$$

They are equal and opposite.

6)

$$\frac{1}{2}\frac{1}{\mu_0}\int_{all\ space} B^2\, dV = \frac{1}{2}\frac{1}{4\pi\times10^{-7}} \cdot (7.7T)^2 \cdot (27\times10^3)(3\times10^{-4})$$

$$= \underline{191 \text{ MJoules}}$$

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