Solutions

Physics 161
Exam 2A
Wednesday, October 29, 2003

Directions: The exam consists of two parts. The first is multiple choice. In the second part you are to solve two problems plus a bonus; here it is important to show your work/thoughts. The exams will be graded out of 100 points. The point-values of each question are indicated; allocate your time wisely.

Part 1

(1-5pts) If the electrical force of repulsion between two electrons which are separated by a distance d is 9.2×10^{-8} N, what is d?

(a) 0.010 nm
(b) 0.025 nm
(c) 0.050 nm ◯
(d) 0.075 nm

$9.2\times10^{-8}\,N = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{d^2}$ ∴ $d = \left(\frac{9\times10^9\cdot 1}{9.2\times10^{-8}}\right)^{1/2}\cdot(1.6\times10^{-19}) = 5\times10^{-11}\,m = 0.050\,nm$

(2-5pts) Two electrons are at rest. The distance between them is 2 nm. What is the minimum amount of work which must be performed to push the electrons together to a separation of 1 nm? (1 eV = 1.6×10^{-19} J)

(a) 0.42 eV
(b) 0.52 eV
(c) 0.62 eV
(d) 0.72 eV ◯

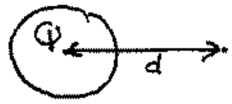
$U_i = \frac{e^2}{4\pi\varepsilon_0 r_i}$, $U_f = \frac{e^2}{4\pi\varepsilon_0 r_f}$, $\Delta U = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{r_f}-\frac{1}{r_i}\right)$

$= 1.6\times10^{-19}\cdot\left[1.6\times10^{-19}\cdot 9\times10^9\cdot\left(\frac{1}{1\times10^{-9}}-\frac{1}{2\times10^{-9}}\right)\right]$

$= 1.6\times10^{-19}\cdot\left[1.6\times9\times\frac{1}{2}\right]\times10^{-1}\cdot(1.6\times10^{-19}) \times 0.72\,J$

(3-5pts) A charge Q is transferred to a metal sphere of radius R. What is the strength of the electric field at a point outside of the sphere, a distance d from the center of the sphere?

(a) $\frac{Q}{4\pi\varepsilon_0 R^2}$
(b) $\frac{Q^2}{4\pi\varepsilon_0 R^2}$
(c) $\frac{Q}{4\pi\varepsilon_0 d^2}$ ◯
(d) $\frac{Q^2}{4\pi\varepsilon_0 d^2}$



$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ where $r = d$

(4-5pts) At one location on the surface of a conductor the charge density is found to be 1 nC/m^2. What is the magnitude of the electric field on the surface, just *outside* of the conductor at this location?

(a) 113 N/C ◯
(b) 57 N/C
(c) 28 N/C
(d) 0 N/C

$E = \frac{\sigma}{\varepsilon_0} = \frac{4\pi\sigma}{4\pi\varepsilon_0} = 9\times10^9\frac{Nm^2}{C^2}\cdot 4\pi\cdot 1\times10^{-9}\frac{C}{m^2} = 4\pi\cdot 9\,N/C = 36\pi\,N/C$

(5-5pts) A spherical shell of radius 500 cm is uniformly charged so that the surface charge density is 1 nC/m^2. What is the *magnitude of the electric field at a point just outside the surface of the shell*?

(a) 113 N/C ◯
(b) 57 N/C
(c) 28 N/C
(d) 0 N/C

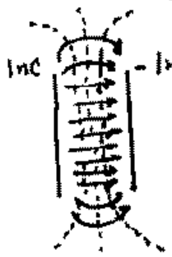
(6-5pts) A metal sphere of radius 5 m is uniformly charged with an excess of 5×10^{10} electrons. What is the surface charge density on the sphere?

(a) -0.015 nC/m^2
(b) -0.025 nC/m^2 ◯
(c) -0.035 nC/m^2
(d) -0.045 nC/m^2

$\frac{5\times10^{10}\times1.6\times10^{-19}\,C}{4\pi\cdot(5m)^2} = \frac{5\times1.6}{4\pi\cdot5\cdot5}\times10^{-9}\,C/m^2 = \frac{1.6}{20\pi}\,nC/m^2 = 0.025\,nC/m^2$

(7-5pts) Two closely spaced parallel metal plates (a parallel plate capacitor) are charged by hooking each to opposite terminals of a battery. The battery is then removed, leaving one of the plates with a positive charge of 1 nC, and the other with a negative charge of -1 nC. Which of the following is true?

(a) The surfaces of equipotential are perpendicular to the plates.
(b) The lines of electric field are parallel to the plates.
(c) The lines of electric field are perpendicular to the equipotential surfaces. ◯



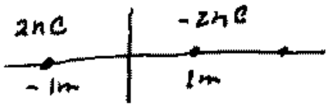
(8-5pts) A uniform electric field has a strength $\vec{E} = 2$ N/C $\hat{\imath}$+1 N/C $\hat{\jmath}$. Consider a square surface with area vector $\vec{A} = 1$ m^2 $\hat{\imath}$ - 2 m^2 $\hat{\jmath}$. What is magnitude of the electric flux through this surface?

(a) 0 Nm^2/C
(b) 1 Nm^2/C
(c) 4 Nm^2/C
(d) 5 Nm^2/C

$\vec{E}\cdot\vec{A} = (2\,N/C\,\hat{\imath} + 1\,N/C\,\hat{\jmath})\cdot(1\,m^2\,\hat{\imath} - 2\,m^2\,\hat{\jmath}) = 2\,Nm^2/C - 2\,Nm^2/C = 0\,Nm^2/C$

(9-5pts) Two point charges are positioned along the x axis. A charge of 2 nC is positioned at $x = -1$ m, and a charge of -2 nC is positioned at $x = 1$ m. What is the electric field on the x axis at $x = 2$ m?

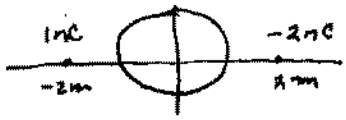
(a) $\vec{E} = 15$ N/C $\hat{\imath}$
(b) $\vec{E} = -30$ N/C $\hat{\imath}$
(c) $\vec{E} = -27$N/C $\hat{\imath}$
(d) $\vec{E} = -16$ N/C $\hat{\imath}$



$E = \frac{2nC}{4\pi\varepsilon_0}\frac{1}{3^2} - \frac{2nC}{4\pi\varepsilon_0}\frac{1}{1^2} = 2nC \times 9\times10^9 \cdot \left(\frac{1}{9} - 1\right) = -\frac{8}{9}\cdot 2\cdot 9\,\frac{N}{C}$

(10-5pts) Two point charges are positioned along the x axis. A charge of 1 nC is positioned at $x = -2$ m ($y = z = 0$), and a charge of -2 nC is positioned at $x = 2$ m ($y = z = 0$). What is the total electrical flux from these charges through a spherical Gaussian surface of radius 1 m which is centered at the origin?

(a) -93 Nm^2/C
(b) -65 Nm^2/C
(c) -23 Nm^2/C
(d) 0 Nm^2/C



$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0} = 0$

(11-5pts) The electric flux through a spherical Gaussian surface having a 2 m radius is 16π Nm^2/C. Which of the following must be true?

(a) The magnitude of the electric field at the Gaussian surface is 1 N/C
(b) *The charge enclosed by the Gaussian surface is 0.44 nC*
(c) The total charge enclosed by the Gaussian surface is zero.
(d) The charge enclosed by the Gaussian surface is located at the center of the sphere.

$16\pi\,\frac{Nm^2}{C} = \frac{Q_{enclosed}}{\varepsilon_0}$

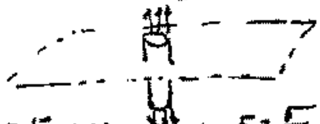
$Q_{enclosed} = (4\pi\varepsilon_0)\cdot 4 = \frac{1}{9\times10^9}\cdot 4 = \frac{4}{9}\times10^{-9}\,C$

(12-5pts) Which is true?

(a) Charge which resides on a conductor must always be on the surface of the conductor.
(b) Lines of equipotential are perpendicular to the surface of a conductor.
(c) *Lines of electric field are parallel to the surface of a conductor.*
(d) Charge on a conductor is uniformly distributed.

(13-5pts) Positive charge is distributed uniformly on an infinite planar surface. Which of the following is true about the *electrical potential* for a point a distance r above the surface?

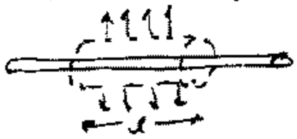
(a) The electrical potential is a constant, independent of r.
(b) The electrical potential ~~increases~~ decreases linearly with the distance r above the surface.
(c) The electrical potential decreases with distance, as $\ln(1/r)$.
(d) The electrical potential decreases with distance, as $1/r$.



$\oint \vec{E}\cdot d\vec{A} = 2EA = \frac{\sigma}{\varepsilon_0}A$ $\quad E = \frac{\sigma}{2\varepsilon_0}$ = const. $\quad \phi = -\frac{\sigma}{2\varepsilon_0}r$

(14-5pts) Positive charge is distributed uniformly on the surface of an infinitely long cylinder. Which of the following is true about the *electric field* for a point at a radius r, outside of the cylinder?

(a) The electric field is constant, independent of r.
(b) The electric field decreases with distance as $\ln(1/r)$.
(c) The electric field decreases with distance as $1/r$.
(d) The electric field decreases with distance as $1/r^2$.

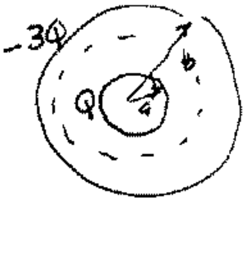


$\oint \vec{E}\cdot d\vec{A} = E\cdot 2\pi r\,l = \frac{\lambda l}{\varepsilon_0}$

$E = \frac{\lambda}{2\pi\varepsilon_0 r}$

(15-5pts) A capacitor consists of two concentric metal spherical shells. The inner shell having radius a is surrounded concentrically by the outer shell having radius b. A charge Q is placed on the inner shell, and three times as much charge of opposite sign, $-3Q$, is placed on the outer shell. Which expression describes the magnitude of the electric field at a radius r between the two shells?

(a) $\frac{Q}{4\pi\epsilon_0 r^2}$.
(b) $\frac{Q}{2\pi\epsilon_0 r^2}$,
(c) $\frac{Q}{\pi\epsilon_0 r^2}$.
(d) $\frac{Q}{4\pi\epsilon_0 r}$



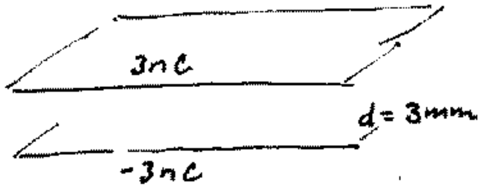
$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$

$E\,4\pi r^2 = \frac{Q}{\varepsilon_0}$

(No influence of the -3Q charge)

Part 2

(1-20pts) A capacitor consists of two closely spaced parallel conducting plates each having area A, separated from one another by a distance of 3 mm. The capacitor is charged so that one plate has a charge of 3 nC, and the other plate has charge of -3 nC. If the potential difference between the plates is 1 V, what is A?



$V = E \cdot d$

$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$

$\therefore V = \frac{Qd}{A\varepsilon_0}$

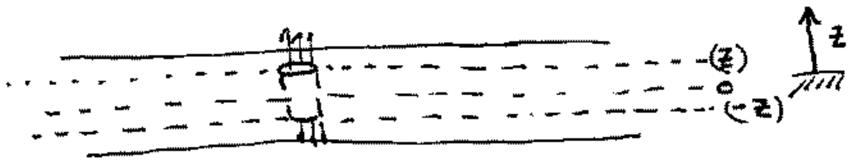
$\therefore A = \frac{Qd}{\varepsilon_0 V} = \frac{3nC \cdot 4\pi \cdot 9\times10^9 \cdot 10^{-3}}{1V}$

$= 3 \cdot 4\pi \cdot 9 \times 10^{-3}\ m^2$

$= 108\pi \times 10^{-3} m^2$

$= 0.34 m^2$

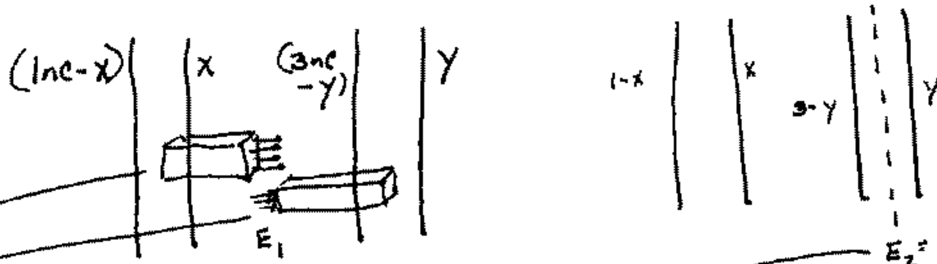
(2-5pts) A non-conducting planar slab of infinite extent has a thickness d. Charge is distributed *uniformly throughout its volume*, with a density ρ. Derive an expression for the electric field *inside the slab*, as a function of the distance from the center of the slab.



$\oint \vec{E} \cdot d\vec{A} = 2E\cancel{A} = \frac{Q_{enclosed}}{\varepsilon_0} = \frac{\rho \cdot \cancel{A} \cdot (2z)}{\varepsilon_0}$

$\therefore \vec{E} = \frac{\rho z}{\varepsilon_0}\hat{z}$

(3-Bonus: 5pts) Two closely spaced parallel metal plates (the configuration of a parallel plate capacitor) are charged. The plate on the left has 1 nC of excess charge. The plate on the right has an excess charge of 3 nC? How does this charge apportion itself between the four surfaces? (How much charge resides on (i) the left surface of the left plate, (ii) the right surface of the left plate, (iii) the left surface of the right plate, and (iv) the right surface of the right plate?)



(Field inside metals must be zero.)

$\oint \vec{E} \cdot d\vec{A} = E_1 A = \frac{X}{\varepsilon_0}$

$E_1 = \frac{X}{\varepsilon_0 A}$

(Field at same point should not depend on choice of Gaussian surface.)

$\oint \vec{E} \cdot d\vec{A} = -E_1 A = \frac{3-Y}{\varepsilon_0}$

$E_1 = \frac{Y-3}{\varepsilon_0 A}$

$\therefore \boxed{X = Y - 3}$

$E_2 = \frac{1-X}{2\varepsilon_0 A} + \frac{X}{2\varepsilon_0 A} + \frac{3-Y}{2\varepsilon_0 A} - \frac{Y}{2\varepsilon_0 A}$

$= \frac{1}{2\varepsilon_0 A} + \frac{3}{2\varepsilon_0 A} - \frac{2Y}{2\varepsilon_0 A} = 0$

$\boxed{4 = 2Y \Rightarrow Y = 2}$

$\therefore Y = 2nC$ and $X = -1nC$

$1-X = \underline{2nC}$, $X = \underline{-1nC}$, $3-Y = \underline{1nC}$, $Y = \underline{2nC}$

2 | | -1 1 | | 2