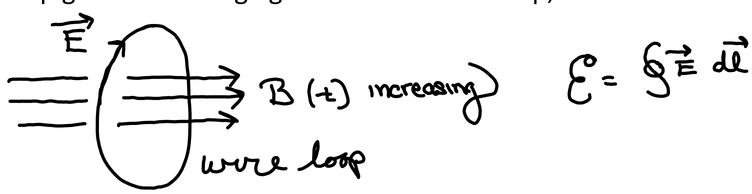
April 17, 2021

Notes for Physics 1320

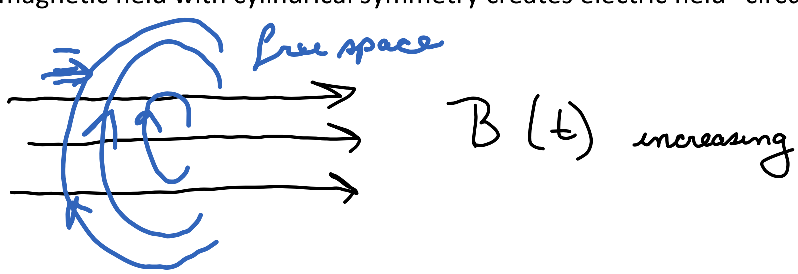
Faraday's Law II

Suppose that we take a circle of wire and place it in a uniform magnetic field, so that there is a nonzero magnetic flux, $\Phi = \vec{B} \cdot \vec{A}$. Now we change the magnetic field while holding the loop fixed in space, and current starts to flow in the loop. The electrons start to move because they are subjected to an electric field that pushes them around the loop, and the integral of this (nonconservative) electric field around the loop gives the voltage gained around the loop, the EMF.



How do we know there is an electric field? One argument is that since the electrons are not moving to begin with, but start moving when the B-field changes, there must be an electric field present that causes them to accelerate. After all, what else could cause them to accelerate if not an electric field? It can't be the magnetic field – a magnetic field only exerts a force on a charge if the charge is already moving.

If there is an electric field causing the electrons to accelerate, this electric field must exist in this region in space whether or not the wire loop is physically present. We can take the loop of wire away, and the electric field will still be there – indeed, a changing magnetic field creates an electric field in free space. In fact, a changing magnetic field with cylindrical symmetry creates electric field "circulation".

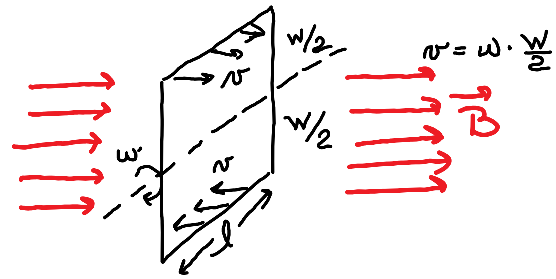


For any radius you choose from the center of the "tube" of magnetic field lines, you can integrate the induced electric field lines around a circle of that radius to find the EMF for a ring of that size. This is what Faraday's law is all about, the generation of an electric field in space by changing a magnetic field in space.

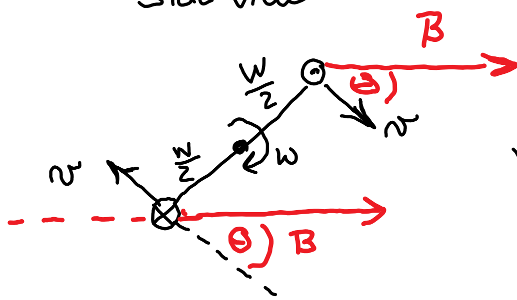
Or is it? Let us return to the AC generator from the previous lecture. In that case, the magnetic field was held constant in space, and the loop of wire was rotated, and that rotation is apparently what generated the current in the loop. Yet if the magnetic field is constant, then here is **no induced E-**field. What causes the electrons to flow? Here we can understand the motion of the electrons from the magnetic force law,

$$\vec{F} = q\vec{v}\times\vec{B}\ ,$$

When we rotate the wire loop we are forcing the electrons to move with velocity that is perpendicular to B at times. Let's consider explicitly the example of a rectangular loop with length ℓ and width w, as shown in the figure below. Focus your attention on the horizontal wires top and bottom – the forces on the vertical wires back and front are perpendicular to the wires and have no effect.



side view



$$|\vec{F}| = qvB\sin\theta$$

$$\text{Work} = 2|\vec{F}|\ell$$
$$= 2qvB\ell\sin\theta$$
$$= 2q\left(\omega\cdot\frac{w}{2}\right)B\ell\sin\theta$$

$$\mathcal{E} = \frac{\text{Work}}{q} = \omega B \underbrace{w\ell}_{\text{Area}} \sin\theta \quad \omega t$$

$$= \omega BA\sin(\omega t)$$
$$= -\frac{d}{dt}\left[BA\cos(\omega t)\right]$$

$$\boxed{\mathcal{E} = -\frac{d}{dt}\left(\vec{B}\cdot\vec{A}\right)}$$

Let me summarize the algebra above: Because the electrons are forced to move in the magnetic field as we rotate the loop, they experience the magnetic velocity-dependent force $q\vec{v}\times\vec{B}$. If we integrate this force around the closed loop of wire, we get the work performed on each electron. The work per charge is the EMF around the loop. You can see that the expression for the EMF has exactly the same form that we found from Faraday's law. (Although we worked this out for a rectangular loop of wire, we can show this same identity for a loop of wire of any shape.)

In summary, when you write

$$\mathcal{E} = -\frac{d}{dt}(\vec{B}\cdot\vec{A})$$

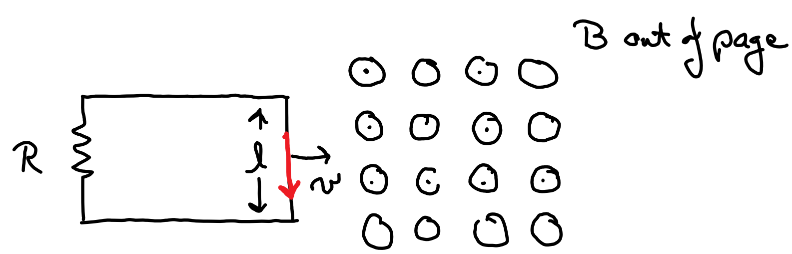
you can generate an emf when B changes and A is constant, or when B is constant and A changes, or a little bit of each. Isn't this all we are saying?

For the rotating loop, the *physical meaning* is different. In the previous case where the B-field changes in time and the area of the loop stays fixed in space, there is an induced electric field that pushes the electrons around the loop. Yet in the present case where the B-field is fixed, and the area of the loop changes in time (because of the rotating loop of wire), there is no induced electric field, and the electrons are pushed around the loop by the magnetic force. We have two apparently different physical reasons for what amounts to the very same physical effect; at the end of the day, the electrons move around the loop, the emf is the same in both cases, and the current is the same in both cases,

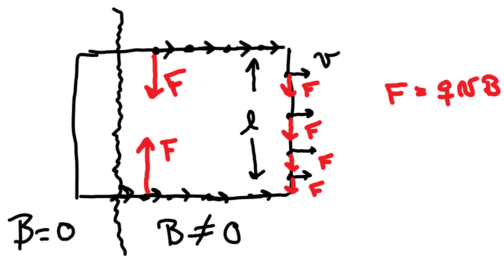
$$I = \mathcal{E}/R$$

For some, it is disconcerting to have different physical mechanism for what amounts to the same physical effect. For example, according to Einstein, the aforementioned disparity caused him to develop the theory of relativity.

Let us look at another example of electrical generation where the magnetic field stays fixed while the wire loop moves. Consider a square loop of wire with resistance R moving to the right with a velocity v into a region in space where there is a B-field perpendicular to v, as shown in the figure.



When the right side of the loop enters the region where the magnetic field is nonzero, the electrons in that part of the loop experience the magnetic force. According to the right hand rule, $\vec{v}\times\vec{B}$ is down, so electrons will be driven down the loop, in the direction indicated by the red arrow. (Remember, except in cases where signs matter, we take the moving charges in wires (electrons) to be positive, so that current flow is in the same direction as particle flow.) Since v is perpendicular to B, we can dispense with the cross product. For the top and bottom sections of the loop, the forces are in the vertical direction, perpendicular to the wire.



The emf around the loop is given by

$$\mathcal{E} = vB\ell$$

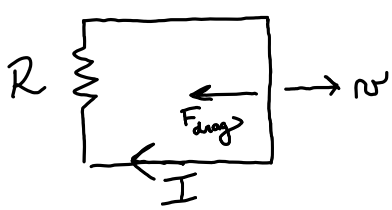
where ℓ is the width off the loop until the left hand side of the wire loop enters the field, and then the EMF goes to zero. (Do you see why?) The current induced in the loop is

$$I = \mathcal{E}/R = \frac{vB\ell}{R}$$

Yet, once the current starts flowing in the loop, there is a force on the right hand wire given by $\vec{F} = I\vec{\ell} \times \vec{B} \to I\ell B$ and this force opposes the velocity. This is your Lenz's Law "back reaction" – the system responds in such a manner as to prevent the loop of wire from entering the field. Putting in the epression for the current above, we have

$$F_{drag} = \left(\frac{v\ell B}{R}\right)\ell B = \frac{(\ell B)^2}{R} v$$

You will want to check with your right hand to convince yourself that this force acts like a frictional drag force, opposing the velocity.



Notice that the drag force is proportional to the velocity, so this imitates Stoke's Law damping.

Let examine the class demonstration that illustrates this "magnetic damping" effect. A large pendulum is fitted with an aluminum plate at the bottom. When the plate passes through the poles of a magnet, currents are induced. The back-reaction to this currents is to slow the pendulum down. The difference between this and the discussion above is that there isn't a particular pathway, or loop, for the electrons in the aluminum to follow. It turns out that the electrons follow an egg-beater pattern, the flow being called "eddy currents".

Let us start by recalling the standard usage, which refers to the currents that are set up when a river flows around an obstacle, like a large boulder, or a protrusion from the shore. The water below the obstacle circulates around and flows back upstream before bending out and rejoining the main flow. Apparently this is a good place to look if you are panning for gold.

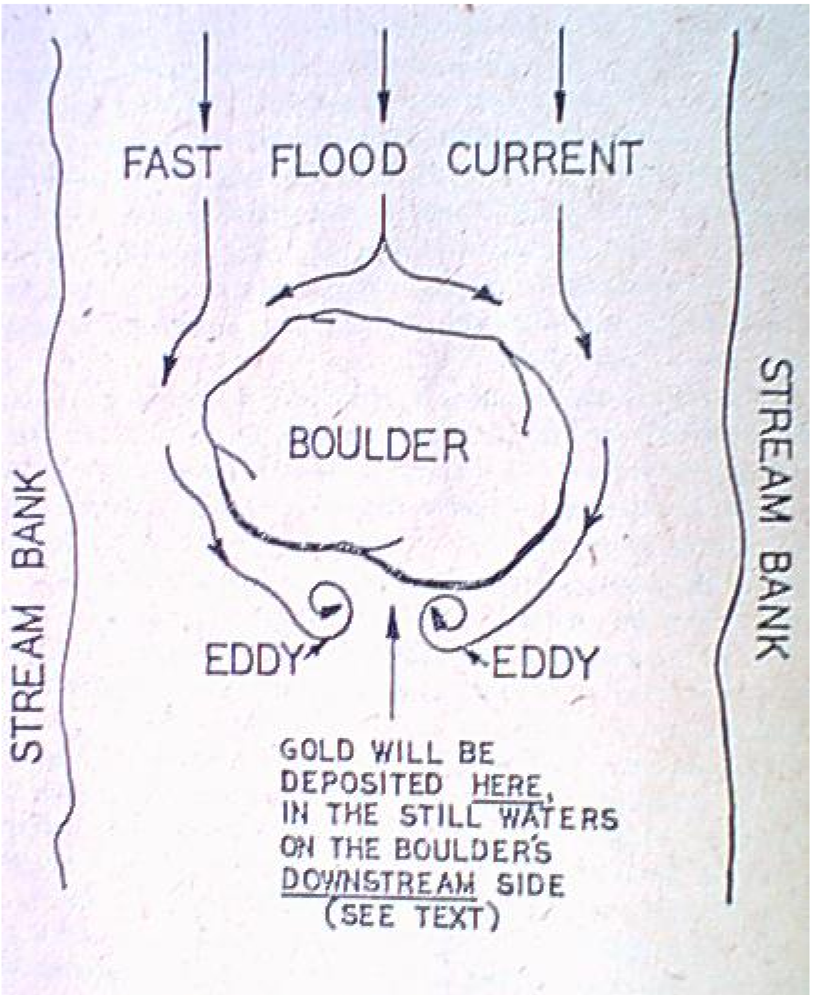
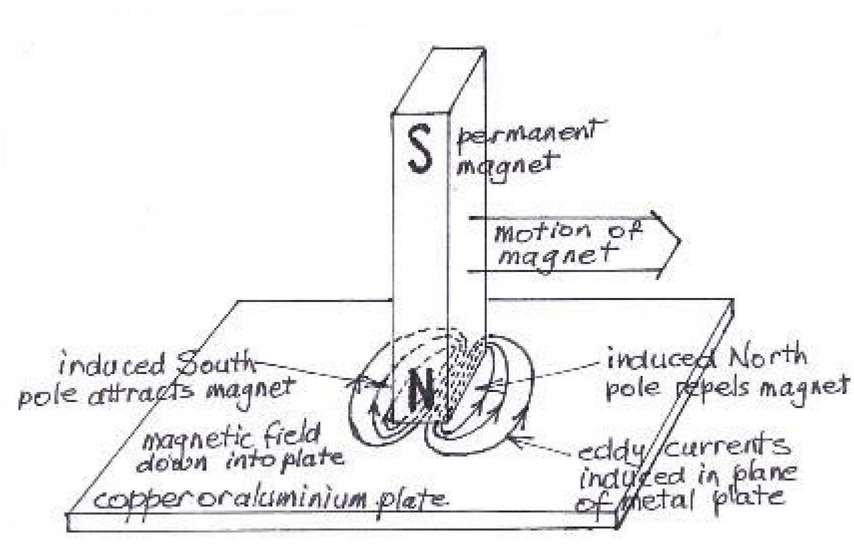


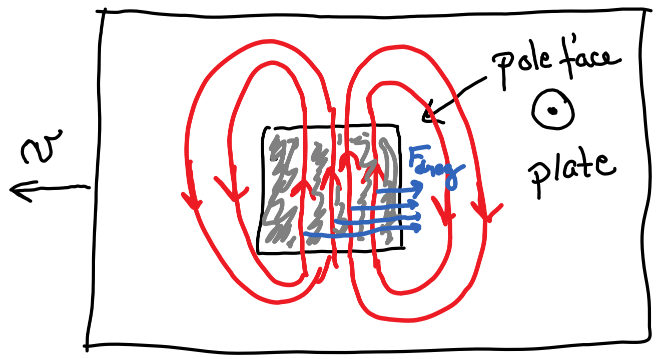
Figure 27—The deposition of gold around a boulder.

If you have the opportunity to paddle a boat up behind a rock like this, you will find that the boat is "sucked" back up behind the rock in the eddy current flow, and if the eddy is large enough you can maintain a stationary position for a good long time. Paddlers who cannot get to shore will often use eddys as a place to rest (or simply as a way to prolong their river adventure).

In our case, when a magnet moves over a plate, egg-beater currents will move above the leading and trailing edge of the magnet. See the figure below.

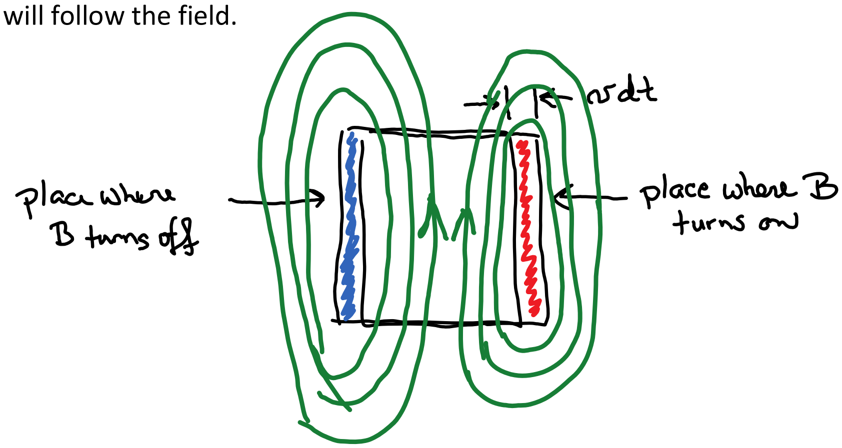


If we think of the magnet as stationary and the metal plate as moving, which is the scenario in our demonstration, then electrons in the region where there is a magnetic field will experience the magnetic deflection $q\vec{v} \times \vec{B}$ and be driven transverse to the direction of the plate's motion. This must be accompanied by return-currents outside of the magnetic field.



The back-reaction comes from the fact that the vertically-moving current in the region within the magnet's pole face experiences a drag force $I\vec{\ell}\times\vec{B}$ that, for the figure above, acts to the right, opposing the velocity of the plate.

Another way to appreciate the eddy currents is to put yourself in the rest frame of the moving plate. From your perspective, the magnet is moving by with a speed v, and you are stationary. As a result of the changing magnetic field, an electric field is induced in space. The only place the magnetic field changes in time is at the leading edge and the trailing edge of the magnet, and these are like two "slivers" of changing flux. The induced electric field will encircle these two slivers, and the current in the metal will follow the field.



The pattern that you predict for the eddy currents should be independent of your choice of reference frame, although it appears that the "physics" is different for the two frames.