Homework 14

Due: Friday, May 6th, 2022

Reminder: The Makeup Exam will be on Wednesday evening, from 7:30-9:30 pm, in Regener Hall Rm 103.

Read Chapter 31.2-31.5 on driven RLC series circuits.

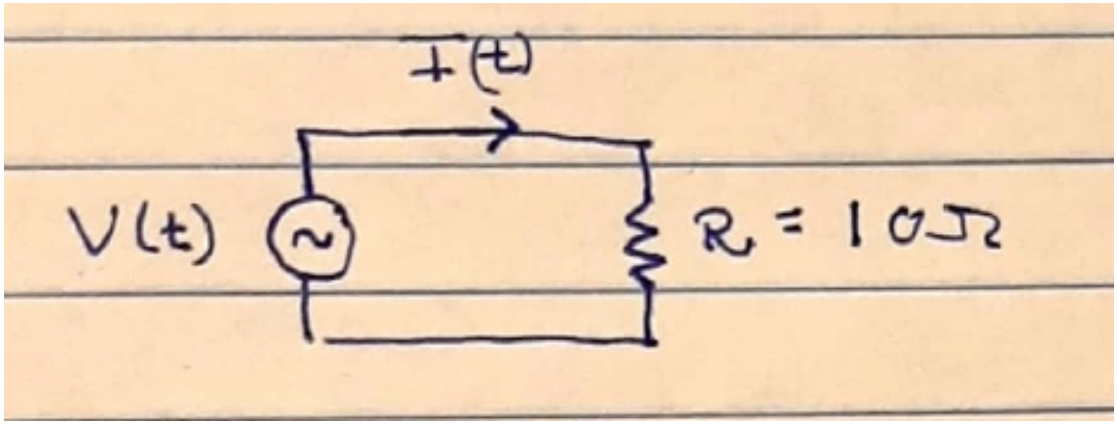
If you are interested in reading about the competion between Westinghouse and Edison as to whether the US should be powered by AC or by DC electricity, here is an article from Smithsonian magazine. https://www.smithsonianmag.com/history/edison-vs-westinghouse-a-shocking-rivalry-102146036/

Make use of $\mathcal{E}LI$ the $IC\mathcal{E}$ man to complete the first 4 problems.

1. A sinusoidal ac voltage source $V(t)$ oscillates with an amplitude $\mathcal{E} = 170$ volts and a frequency $\omega = 377$ rad/s (60 Hz). The source is connected in series in a closed loop with 10 Ω resistor. The current through the resistor is given by $I(t) = I_0 \cos \omega t$ in steady state. Apply Kirchoff's voltage law,

$$V - IR = 0,$$

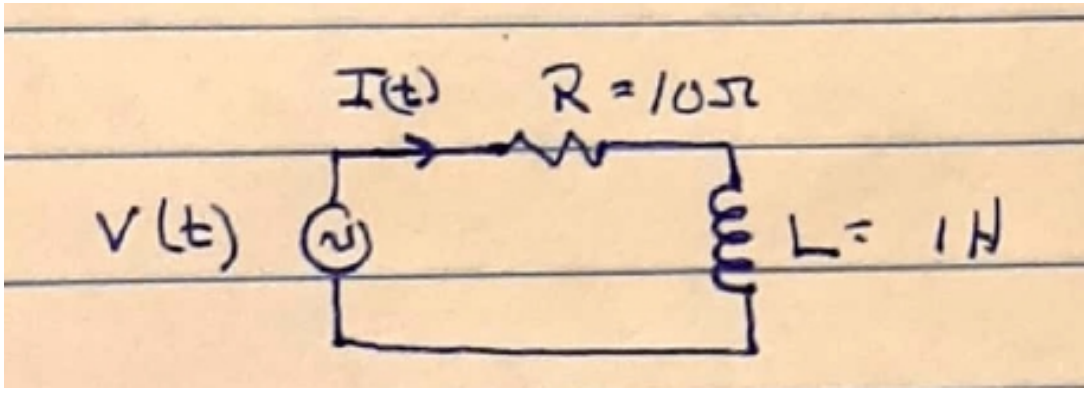
find the current amplitude I_0, and find an expression for $V(t)$. What is the average power $\overline{V(t)I(t)}$ delivered to the circuit?



2. A sinusoidal ac voltage source $V(t)$ oscillates with an amplitude $\mathcal{E} = 170$ volts and a frequency $\omega = 377$ rad/s (60 Hz). The source is connected in series in a closed loop with a 1 H inductor. The current through the inductor is given by $I(t) = I_0 \cos \omega t$ in steady state. Apply Kirchoff's voltage law,

$$V - IX_L^{(\uparrow)} = 0,$$

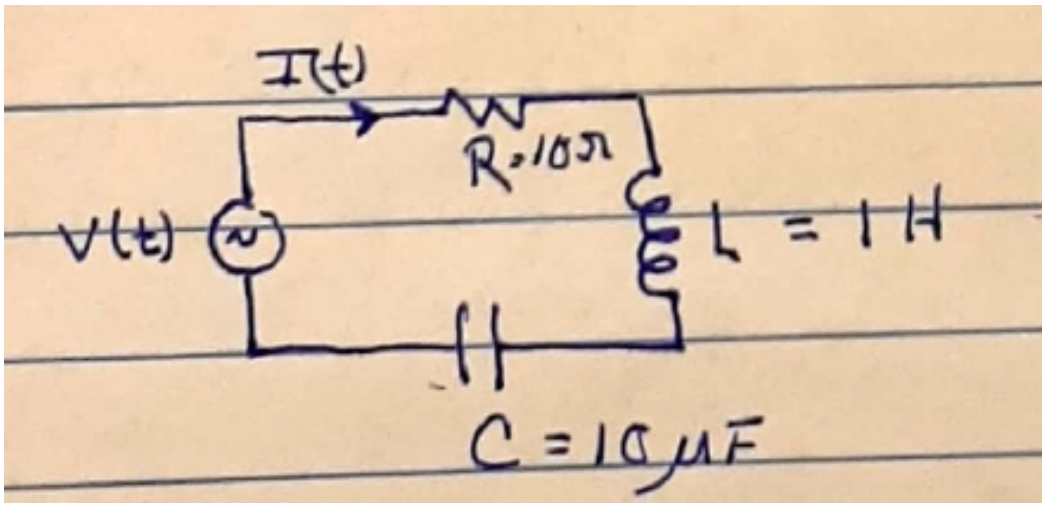
find the inductive reactance X_L, find the current amplitude I_0, and write down an expression for the source voltage $V(t)$. What is the average power $\overline{V(t)I(t)}$ delivered to the circuit?



3. A sinusoidal ac voltage source $V(t)$ oscillates with an amplitude $\mathcal{E} = 170$ volts and a frequency $\omega = 377$ rad/s (60 Hz). The source is connected in series in a closed loop with a 10 Ω resistor, a 1 H inductor, and a 10 μF capacitor. The current through the capacitor is given by $I(t) = I_0 \cos \omega t$ in steady state. Apply Kirchoff's voltage law,

$$V - IR - IX_L^{(\uparrow)} - IX_C^{(\downarrow)} = 0,$$

find the overall impedance Z, find the current amplitude I_0, find the phase angle ϕ, and write down an expression for the source voltage $V(t)$. What is the average power $\overline{V(t)I(t)}$ delivered to the circuit?



4. Carry out the calculations for problem 3 above for an arbitrary driving frequency ω, and make a graph of the average power $\overline{V(t)I(t)}$ delivered to the circuit versus ω, in the range 200 rad/s to 400 rad/s. Your graph should show a peak at the resonant frequency.

5. Below are the four Maxwell equations.

$$(i)\ \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

$$(ii)\ \oint \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

$$(iii)\ \oint \vec{B} \cdot d\vec{A} = 0$$

$$(iv)\ \oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enclosed}} + \mu_0 \epsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$

Memorize these equations. What are the names/people associated with these four laws? Which of the two originated from Coulomb's law? Which of the two originated from the Biot-Savart law? Which is known as Ampere's law? Which is Gauss's law? Which is sometimes called "Gauss's law for magnetic fields?" Which is Faraday's law? Which of the four laws was modified by Maxwell on purely theoretical grounds?

5. Electromagnetic waves travel at a speed given by $\sqrt{\frac{1}{\mu_0 \epsilon_0}}$ in free space. Insert the values of μ_0 and ϵ_0 and calculate the speed in meters per second (and also in miles per second). How close is this to the speed of light? If you would like to learn more about Maxwell and his discovery of electromagnetic waves, the world's expert on this subject is Malcolm Longair (https://en.wikipedia.org/wiki/Malcolm_Longair). Here is an excellent talk on Maxwell, delivered by Longair in 2021.

https://www.youtube.com/watch?v=2H4ycW6BgoQ