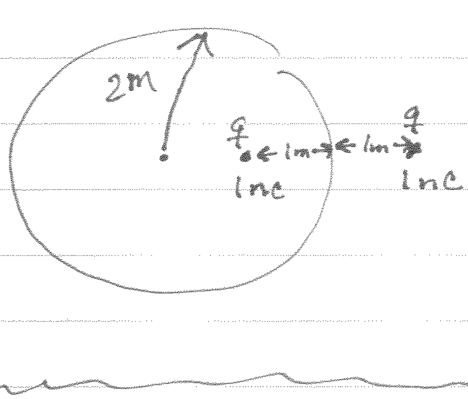
Flashcards: Part 4.

The electric field is $\vec{E} = 5\frac{N}{C}\hat{\imath}$. The area of an imaginary rectangle of dimensions 5 m by 3 m points in the direction of the unit vector $\frac{\hat{\imath}-\hat{\jmath}}{\sqrt{2}}$. What is the flux through this rectangle?

$$\int \vec{E}\cdot d\vec{A} = \vec{E}\cdot\vec{A}$$
$$= \left(\frac{5N}{C}\right)(15m^2)\ \hat{\imath}\cdot\frac{(\hat{\imath}-\hat{\jmath})}{\sqrt{2}}$$
$$= 75\frac{Nm^2}{C}\cdot\frac{1}{\sqrt{2}}$$

An imaginary spherical surface with radius $r = 2m$ surrounds a charge $q = 1nC$. Outside of the sphere is a second identical charge. What is $\oint_{\text{sphere}} \vec{E}\cdot d\vec{A}$?

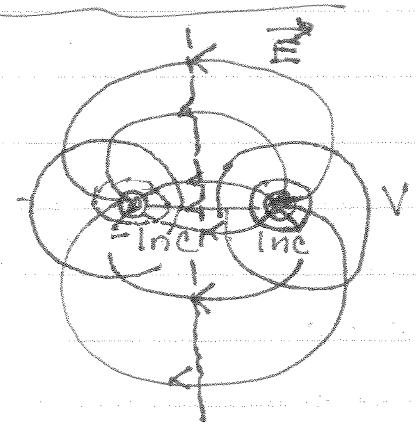


$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$$
$$= \frac{1nC\cdot 4\pi}{4\pi\varepsilon_0}$$
$$= 4\pi \times 10^{-9} C \cdot 9\times 10^{9}\frac{Nm^2}{C^2}$$
$$= \underline{36\pi\ Nm^2/C}$$

How does your answer to the problem above change if the charge inside is $-1nC$?

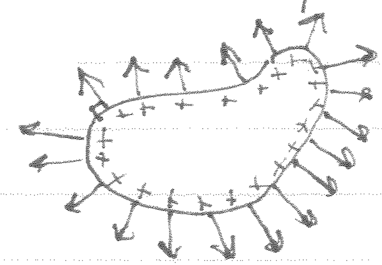
$$\oint \vec{E}\cdot d\vec{A} = -36\pi\frac{Nm^2}{C}.$$

For the problem above, sketch the electric field lines and the lines indicating equipotential surfaces.

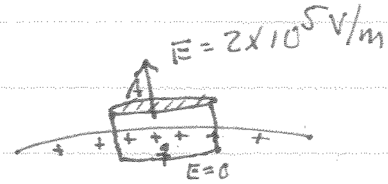


What is special about the lines of $\vec{E}$ coming from a charged conducting surface (in equilibrium).

They come away $\perp$ to surface.



At a point on the outside surface of a charged conductor the electric field is 2×10^5 V/m. What is the charge density on the metal surface at that location?



$$E = \frac{q}{\varepsilon_0 A} = \frac{\sigma}{\varepsilon_0}$$

$$\sigma = \varepsilon_0 \cdot 2\times10^5\,\text{V/m}$$

$$= \frac{1}{4\pi} 4\pi\varepsilon_0 \cdot 2\times10^5\,\text{V/m}$$

$$= \frac{1}{4\pi} \frac{1}{9\times10^9} \cdot 2\times10^5\,\text{V/m}$$

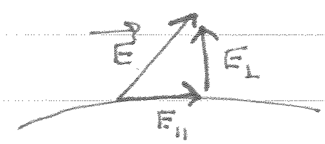
$$= \frac{2}{4\pi \cdot 9}\times10^{-4}\,\frac{V}{m}\cdot\frac{C^2}{N m^2}$$

$$= \frac{1}{18\pi}\times10^{-4}\,C/m^2$$

Why do we know that it must be that $\vec{E}=0$ everywhere inside a conductor in equilibrium?

If $\vec{E}$ were not zero, then $\vec{J}=ne\mu\vec{E}$ would not be zero, and we don't believe in perpetual motion.

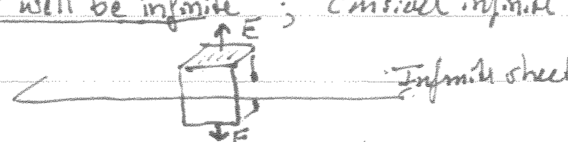
Why does $\vec{E}$ have to be $\perp$ to the surface of a conductor in equilibrium?



If $\vec{E}$ came away from a metal surface at an angle, there would be a component of $\vec{E}$ parallel to the surface, causing perpetual surface currents.

A charge of 1 Coulomb is distributed uniformly on ~~surface~~ planar a flat disk with radius 1 meter. How strong is the electric field just above the surface in the middle?

Close to the surface, the <u>disk may as well be infinite</u>; Consider infinite case:



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0} \Rightarrow 2EA$$

propositions:

$$E = \left(\frac{q_{enclosed}}{A}\right)\frac{1}{2\varepsilon_0}$$

$$\frac{q_{enclosed}}{A} = \frac{1\,\text{coulomb}}{\pi(1m)^2} \quad \therefore \quad E = \frac{1c}{\pi(1m)^2}\cdot\frac{4\pi}{2}\cdot 9\times10^9\frac{Nm^2}{c^2}$$

$$= 18\times10^9\ N/C$$

A sphere with radius $R = 1\text{m}$ has a uniform surface charge of 1 Coulomb. What is $\vec{E}$ just outside the sphere?

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{r}$$

$$= \frac{1\,\text{C} \cdot 9\times10^9 \frac{\text{Nm}^2}{\text{C}^2}}{(1\text{m})^2}\hat{r}$$

$$= 9\times10^9 \frac{\text{N}}{\text{C}}\hat{r}$$

What is $\vec{E}$ inside the sphere?

$\vec{E} = 0$ for $r < R$.

Consider taking 10 nC and putting some of the charge on the surface of a sphere with radius 1m, and the rest of it on the surface of a sphere with radius 0.25 m. If the spheres are far apart, what is the configuration with the lowest energy?

$Q = 10\,\text{nC}$

$C_1 = 4\pi\varepsilon_0 R_1$ $\qquad$ $C_2 = 4\pi\varepsilon_0 R_2$

(capacitance of isolated sphere)

$$U = \frac{(Q-q)^2}{2C_1} + \frac{q^2}{2C_2}$$

$$\frac{dU}{dq} = -\frac{(Q-q)}{C_1} + \frac{q}{C_2} = 0$$

$$q = \frac{Q/C_1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

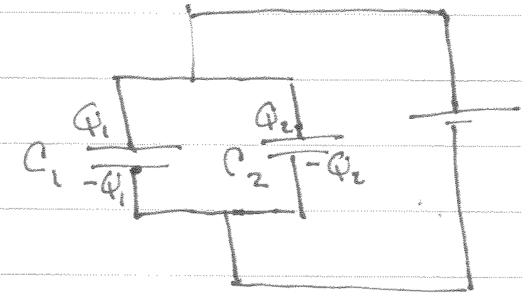
$$= \frac{Q\,C_2}{C_1 + C_2} = \frac{Q\,R_2}{R_1 + R_2}$$

$$= 10\,\text{nC} \cdot \frac{(0.25)}{(1.25)}$$

$$q = 2\,\text{nC}.$$

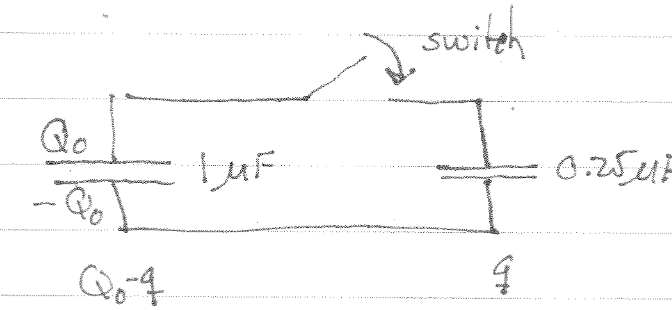
$$Q - q = 8\,\text{nC}$$

Consider two capacitors C_1 and C_2 connected in $\parallel$ to a 9V battery. How much charge will be on each capacitor in equilibrium?



$$\boxed{\begin{aligned} Q_1 &= C_1 \cdot 9V \\ Q_2 &= C_2 \cdot 9V \end{aligned}}$$

A $1\mu F$ capacitor is initially charged to $10\,nC = Q_0$. When the switch is closed, some of this charge moves to the $0.25\mu F$ capacitor. How will the charge be distributed in equilibrium?



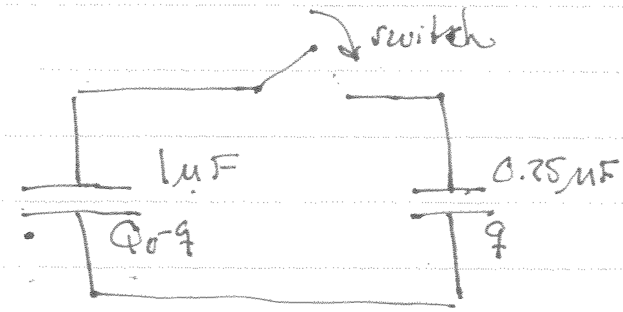
$$U = \frac{(Q_0 - q)^2}{2 \cdot 1\mu F} + \frac{q^2}{2 \cdot (0.25\mu F)}$$

$$\frac{dU}{dq} = -\frac{(Q_0 - q)}{1\mu F} + \frac{q}{0.25\mu F} = 0$$

$$q\left(\frac{1}{.25} + \frac{1}{1}\right) = \frac{Q_0}{1}$$

$$\left\{ \begin{aligned} q &= \frac{0.25\,Q_0}{1.25} = \frac{1}{5} Q_0 = \underline{\underline{2\,nC}} \\ Q_0 - q &= \underline{\underline{8\,nC}} \end{aligned} \right.$$

A $1\mu F$ capacitor is initially charged to $10nC = Q_0$. When the switch is closed, some of this charge moves to the $0.25\mu F$ capacitor. How will charge be distributed in equilibrium?



In equilibrium, the two top plates will be at the same potential, and the two bottom plates will be at the same potential. So the voltage drops ΔV across each will be the same.

$$\frac{Q_0 - q}{1\mu F} = \Delta V = \frac{q}{0.25\mu F}$$

$$q\left(1 + \frac{1}{.25}\right) = Q_0$$

$$q = \frac{.25}{1.25} Q_0$$

$$= 2nC$$

$$Q_0 - q = 8nC.$$

An electric field $\vec{E} = 5\frac{N}{C}(\hat{i} - \hat{j} + 3\hat{k})$ is constant. How much work will you have to perform to move a charge of $0.1\mu C$ from $(x,y,z) = (1m, 2m, 3m)$ to $(x,y,z) = (2m, -4m, 1m)$?

$$\Delta V = -\int_{pt1}^{pt2} \vec{E} \cdot d\vec{l}$$

$\vec{E}$ constant

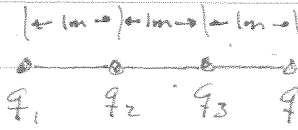
$$\Delta V = -\vec{E} \cdot \vec{l}$$

$$= -\frac{5N}{C}(\hat{i} - \hat{j} + 3\hat{k}) \cdot ((2-1)\hat{i} + (-4-2)\hat{j} + (1-3)\hat{k})$$

$$= -5 J/C(1 + 6 - 6) = -5 J/C$$

$$W = q\Delta V = (-5 J/C)(0.1\mu C) = \underline{\underline{-0.5\mu J}}$$

Consider four point charges each with charge $q = 1\,nC$ equally spaced on the x axis at $x = 0$, 1m, 2m, and 3 m. What minimum amount of work had to be performed to put them there?



$$W = \frac{q_1 q_2}{4\pi\varepsilon_0 r} + \frac{q_3 q_1}{4\pi\varepsilon_0 (2r)} + \frac{q_3 q_2}{4\pi\varepsilon_0 r}$$

$$+ \frac{q_4 q_1}{4\pi\varepsilon_0 (3r)} + \frac{q_4 q_2}{4\pi\varepsilon_0 (2r)} + \frac{q_4 q_3}{4\pi\varepsilon_0 r}$$

$$= \frac{q^2}{4\pi\varepsilon_0 r}\left[1 + \frac{1}{2} + 1 + \frac{1}{3} + \frac{1}{2} + 1\right]$$

$$= \frac{q^2}{4\pi\varepsilon_0 r}\left[3 + 2\cdot\left(\frac{1}{2}\right) + 1\cdot\left(\frac{1}{3}\right)\right]$$

$$= \frac{(9\times10^9)(10^{-9})^2}{1}\left[4\frac{1}{3}\right]$$

$$= (9)\left(4\tfrac{1}{3}\right)(10^{-9})\ J.$$