Flashcards : Part 3

The molecular polarizability of the molecules in an insulator describes what?

$\vec{p} = \alpha \vec{E}$ ← molecular polarizability

The proportionality constant between the electric field and the dipole moment induced by the field.

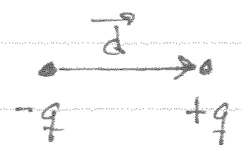
In general, which is more polarizable, a large molecule or a small molecule?

Large.

[A good estimate for α: $\alpha = 4\pi\varepsilon_0 a^3$ where a = molecular radius.]

A dipole moment p is produced when electrons in a molecule are pushed to one side, creating neg. q on one side and pos. q on the other. How is $\vec{p}$ defined?

$\vec{p} = q\vec{d}$



($\vec{d}$ points from negative to positive).

What is a "permanent" dipole moment?

A dipole moment that exists in a molecule naturally, in the absence of an applied field.

Name a molecule that has a permanent dipole moment?

H_2O
CO
NaCl

What is the "bulk polarization" of a material?

$\vec{P} = n\vec{p}$ ← dipole moment of each molecule.
↑ # of molecules / m^3

How is the bulk polarization of a sample related to the applied electric field?

$\vec{P} = \chi \vec{E}$
↑ the "susceptibility"

Why is χ called "susceptibility"?

It tells how susceptible a material is to becoming polarized!

Couldn't they have come up with a better name for χ?

No!

Why do I have to know this "stuff"?

... this is beyond my expertise ....
[Its called science. (?)]

How is susceptibility related to molecular polarizability?

$\chi = n\alpha$

What is the permitivity of free space? — ε_0

What is the permitivity of a material? — ε

How is ε related to ε_0 through the susceptibility χ? — $\varepsilon = \varepsilon_0 + \chi$

What is the "dielectric constant" of an insulator? — $K = \frac{\varepsilon}{\varepsilon_0}$

What is K for plastic (typical)? — $K \cong 3$

What is K for water? — $K \cong 80$

What is K for a molecular solid with molecular density n, and molecular polarizability α?

(i) $p = \alpha E$

(ii) $np \equiv P = n\alpha E \equiv \chi E$

(iii) $\therefore \chi = n\alpha$

(iv) $\varepsilon \equiv \varepsilon_0 + \chi = \varepsilon_0 + n\alpha$

(v) $\frac{\varepsilon}{\varepsilon_0} = \boxed{1 + \frac{n\alpha}{\varepsilon_0} = K}$

If $K = 80$ and $n = 10^{28}\ m^{-3}$, what is α?

$$\alpha = \frac{(K-1)\varepsilon_0}{n}$$

$$= \left[\frac{(79)}{10^{28}\ m^{-3}} \cdot \frac{1}{4\pi(9\times10^9)}\right] \frac{C^2 m}{N}$$

Why do capacitors have dielectrics between the plates rather than just vacuum?

You need something to keep the plates apart, and it has to be insulating.

Why else?

The dielectric reduces the electric field between the plates (for the same amount of charge)

Why does a reduction in field give a larger capacitance?

(i) Lower field means lower ΔV between plates, since $\Delta V_{12} = -\int_1^2 \vec{E}\cdot d\vec{l}$.

(ii) since $C = \frac{Q}{\Delta V}$, smaller ΔV means <u>larger C</u>.

About how large would the capacitance be for two $8\frac{1}{2}'' \times 11''$ sheets of aluminum foil separated by $10\,\mu m$ thick sheet of wax paper?

$\varepsilon \cong 4\varepsilon_0$ wax paper

$$C = \frac{\varepsilon A}{d} = \frac{4\cdot(4\pi\varepsilon_0)}{4\pi}\cdot\frac{A}{d}$$

$$= \frac{1}{\pi}\frac{1}{\left(9\times10^9\,\frac{Nm^2}{C^2}\right)}\cdot\frac{(.3m)^2}{10^{-5}m}$$

($(.3m)^2 \leftarrow 8\frac{1}{2}'' \times 11''$; $10^{-5}m \leftarrow 10\,\mu m$)

$$\approx \frac{10^{-9}\cdot10^{-2}\cdot10^{5}}{\pi} \cong \underline{0.3\,\mu F}$$

If a capacitor with capacitance $C = 0.3\,\mu F$ were charged to a voltage of 1.5 Volts, how much charge would have been transferred from the low voltage plate to the high voltage plate?

$$C = \frac{Q}{\Delta V}$$

so $Q = (0.3\,\mu F)(1.5\,V)$

$= .45\ \mu$ Coulombs

How many electrons are in $0.45\ \mu$ Coulombs?

$$0.45 \times 10^{-6} \text{ Coulombs} \times \frac{1 \text{ electron}}{1.6 \times 10^{-19} \text{ Coulombs}}$$

$$= \frac{0.45}{1.6} \times 10^{13} \cong \underline{\underline{2.5 \times 10^{12}}}$$

What is the minimum amount of work required to charge a $0.3\,\mu F$ capacitor to 1.5 Volts?

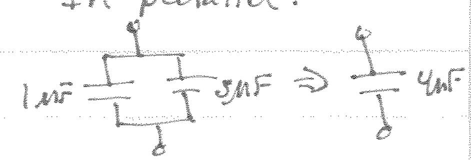
$$\Delta U = \frac{Q^2}{2C} = \frac{(0.45)^2 \times 10^{-12}}{2 \times (0.3 \times 10^{-6} F)}$$

$$\doteq \frac{(0.45)^2}{(0.6)} \times 10^{-6}$$

$$\cong 3 \times 10^{-7}\ J$$

Suppose that you buy two capacitors at Radio Shack, one $3\,\mu F$ and one $1\,\mu F$. How can you connect them to make larger capacitance? (What will be C_{eq}?)

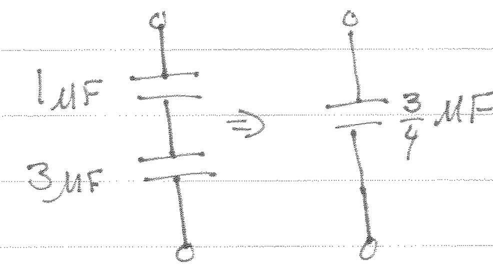
In parallel:



$C_{eq} = C_1 + C_2$

How can you connect 1μF and 3μF to make a smaller capacitor?

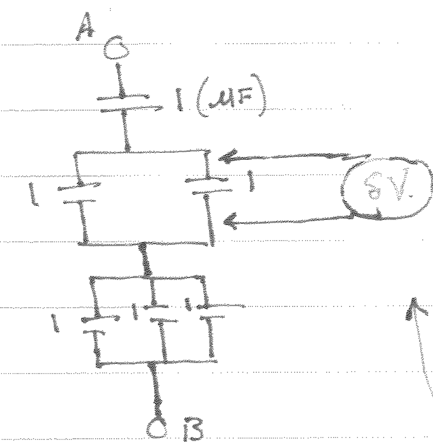
In series:

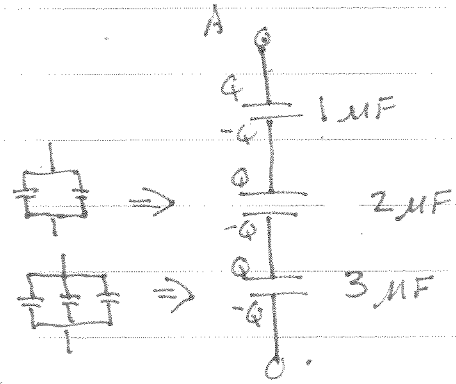


$$\frac{1}{C_{eq}} = \frac{1}{1\mu F} + \frac{1}{3\mu F} = \frac{4}{3\,\mu F}$$

$$C_{eq} = \frac{3}{4}\,\mu F$$

For the capacitors wired together below, what is the equivalent capacitance between A and B?





$$\frac{1}{C_{eq}} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$$

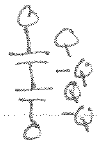
$$C_{eq} = \frac{6}{11}\,\mu F$$

If the voltage between A and B above is 6 Volts, what is the voltage (δV) across the capacitor shown?

$$Q = \left(\frac{6}{11}\,\mu F\right) \cdot 6 \text{ Volts}$$

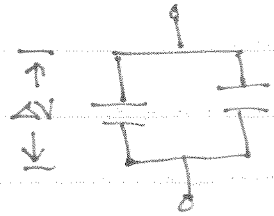
$$= \frac{36}{11}\,\mu \text{ Coulombs.}$$

$$\delta V = \frac{\frac{36}{11}\,\mu \text{ Coulombs}}{2\,\mu F} = \boxed{\frac{18}{11} \text{ Volts}}$$



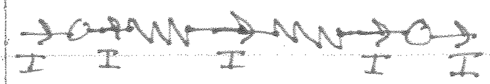
Capacitors in series
all have the same ___?___ Charge

Capacitors in parallel

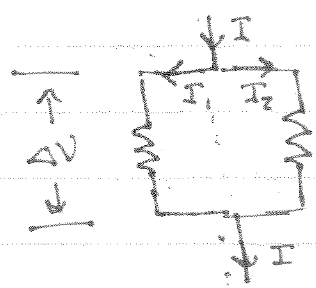


all have the same ___?___ voltage.

Resistors in series
all have the same ___?___ current.



Resistors in parallel
all have the same ___?___ voltage.



Somewhere out there (in free space)
the electric field is 3×10^6 V/m.
What is the energy density
at that location?

$$u = \frac{1}{2}\varepsilon_0 E^2$$

$$= \frac{1}{2}\frac{1}{4\pi} 4\pi\varepsilon_0 E^2$$

$$= \frac{1}{8\pi} \cdot \frac{1}{9\times10^9 \frac{Nm^2}{C^2}} \cdot (3\times10^6 V/m)^2$$

$$= \frac{1}{8\pi} \times 10^3 \; J/m^3$$

The electric field outside of a metal sphere falls off as $E = \frac{3\times10^6\ V\cdot m}{r^2}$

If the radius of the sphere is 1m, what is the energy stored in the field?

$$U = \int_{\text{all space (where } E\neq 0)} \tfrac{1}{2}\varepsilon_0 E^2\, dV$$

$$= \frac{1}{2}\varepsilon_0 (3\times10^6\ Vm)^2 \int_{r=1m}^{\infty} \left(\frac{1}{r^2}\right)^2 4\pi r^2 dr$$

$$= \frac{1}{2}\,\frac{1}{4\pi}\,\frac{1}{(9\times10^9)}\cdot\frac{(3\times10^6\ Vm)^2\, 4\pi}{(1m)}$$

$$= \frac{1}{2}\cdot 10^3 \text{ Joules.}$$

Another way to do problem?

Isolated spherical capacitor:

(i) $C = 4\pi\varepsilon_0 R \qquad R = 1m.$

(ii) $\Delta V = -\int_{r=\infty}^{r=R} \left(\frac{3\times10^6\ V\cdot m}{r^2}\right) dr = -\int_{\infty}^{R} \vec{E}\cdot d\vec{l}$

$= 3\times10^6\ V$

(iii) $Q = C\,\Delta V = (4\pi\varepsilon_0 R)(3\times10^6\ V)$

(iv) $U = \frac{Q^2}{2C} = \frac{(4\pi\varepsilon_0 R)^2 (3\times10^6\ V)^2}{2\,(4\pi\varepsilon_0 R)}$

$= \frac{(3\times10^6\ V)^2\cdot(4\pi\varepsilon_0 R)}{2} = \frac{9\times10^{12}\ V^2\cdot 1m}{2\,(9\times10^9\ \frac{Nm^2}{C^2})}$

$= \frac{1}{2}\times10^3\ J \quad$ (same)