Flashcards: Part 2
continued

If an isolated metal sphere is at a potential of 100,000 V (relative to 0 V at infinity), how small can the radius be? (In dry air.)

conductor: $E_{surface} = \frac{\sigma}{\varepsilon_0} < 3\times10^6\ V/m$

$\sigma = \frac{Q}{4\pi R^2}$

$\Delta V = \frac{Q}{4\pi\varepsilon_0 R}$

$3\times10^6\ V/m > \frac{1}{\varepsilon_0}\cdot\frac{4\pi\varepsilon_0 R\cdot\Delta V}{4\pi R^2} = \frac{\Delta V}{R}$

$R > \frac{100{,}000\ V}{3\times10^6\ V/m} = \frac{1}{3}\times10^{-1}\ m$

$= 3.3\ cm$

What is the "dielectric strength" of dry air?

$\simeq 3\times10^6\ V/m.$

What is the relationship between dielectric constant and dielectric strength?

Not much. Dielectric constant has to do with polarizability at low fields. "Dielectric strength" is a synonym for "threshold electric field".

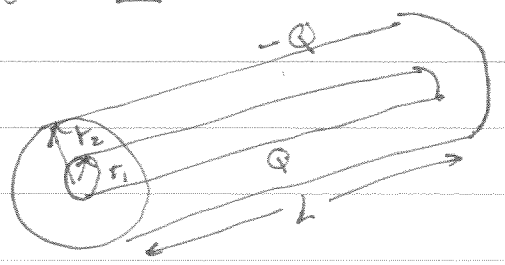
What is the dielectric strength of plastic?

$\simeq 10^8\ V/m$

How strong does E have to be to cause lightning?

$\simeq 3\times10^6\ V/m$

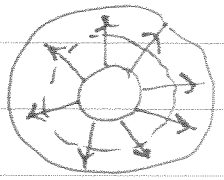
Scene III



A capacitor made of concentric cylinders is charged. The spacer material has permitivity ε.

The cylinders are long enough so that you can ignore fringe fields at the ends.

If charge Q is on the inner cylinder and charge -Q is on the outer cylinder, what is $E(r<r_1)$, $E(r)$ for $r_1<r<r_2$, and $E(r>r_2)$?



$$\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon} = E\cdot 2\pi r\cdot L$$

$$E = \frac{Q/L}{2\pi\varepsilon r} \quad \text{for } r_1<r<r_2$$

$E = 0$ otherwise.

What is ΔV?

$$\Delta V = -\int_{r_2}^{r_1} \vec{E}\cdot d\vec{r}$$

$$= -\frac{Q}{2\pi\varepsilon L}\int_{r_2}^{r_1} \frac{dr}{r}$$

$$= \frac{Q}{2\pi\varepsilon L}\ln(r_2/r_1)$$

What is C?

$$C = \frac{Q}{\Delta V} = \frac{2\pi\varepsilon L}{\ln(r_2/r_1)}$$

What is Ohm's law? $V = IR$

What is the resistivity of a material with $n = 10^{29} \frac{\text{electrons}}{m^3}$ and a mobility of $100 \frac{(cm)^2}{Vs}$?

$$\rho = \frac{1}{ne\mu}$$

$$= \frac{10^{-29} m^3}{1.6 \times 10^{-19} C} \cdot \frac{}{10^{-2} \frac{m^2}{Vs}}$$

$$= 0.6 \times 10^{-8} \Omega m$$

What is the resistance of a leaky parallel plate capacitor with $n = 10^{21} m^{-3}$, $\mu = 10^{-4} \frac{m^2}{Vs}$, $A = 0.01 m^2$ and $L = .01 m$?

$$R = \frac{\rho L}{A} = \frac{1}{10^{21} \cdot (1.6 \times 10^{-19}) 10^{-4}} \cdot \left(\frac{.01}{.01}\right)$$

$$= 100 \Omega$$

What is the resistance of a wire with $n = 10^{29} m^{-3}$, $\mu = 10^{-2} \frac{m^2}{Vs}$, and diameter .4 cm, and length 161 Km (100 miles)?

$$R = \frac{\rho L}{A} = \frac{1}{10^{29} (1.6 \times 10^{-19}) 10^{-2}} \cdot \frac{161 \times 10^3}{\pi (.002)^2}$$

$$= \frac{161}{1.6} \cdot \frac{1}{\pi} \cdot \frac{1}{4} \times 10^{-29+19+2+3+6}$$

$$\simeq 10^2 \Omega$$

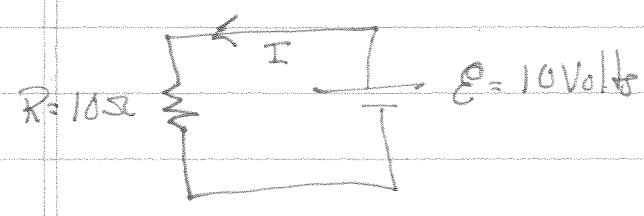
What is the resistance of a leaky capacitor C with resistivity ρ?

$$I = \int_{x\text{-surface}} \vec{J}\cdot d\vec{A} = ne\mu \int \vec{E}\cdot d\vec{A}$$

$$= ne\mu \frac{Q}{\varepsilon} = ne\mu \frac{1}{\varepsilon}\cdot C\Delta V$$

$$R = \frac{\varepsilon}{ne\mu C} = \boxed{\frac{\rho\varepsilon}{C}}$$

What is the current?



$$I = \frac{V}{R} = \frac{10\ \text{Volts}}{10\,\Omega} = 1\ \text{Amp.}$$

If a current of 1A is in a wire with resistivity $\rho = 10^{-8}\,\Omega m$, what is the drift velocity if $n = 10^{29}\,m^{-3}$? and radius is 1mm.

$$I = \vec{J}\cdot\vec{A}$$

$$= ne\vec{v}\cdot\vec{A}$$

$$\Rightarrow nevA$$

$$v = \frac{1\ \text{Amp}}{10^{29}\cdot(1.6\times10^{-19})\,\pi\,10^{-6}\,m^2}$$

$$= \frac{10^{-4}}{(1.6)\pi} \simeq 2\times10^{-5}\ m/s$$

If $v = 10^{-4}\,m/s$, $n_e = 10^{29}\,m^{-3}$, what is the current density?

$$J = nev = 10^{29}\times(1.6\times10^{-19})\,10^{-4}$$

$$= 10^6\ \frac{\text{Coulombs}}{\text{sec}}\ \frac{1}{m^2}$$

If $v = 10^{-4}\,m/s$, $n_e = 10^{29}\,m^{-3}$, what is the current in a wire with $r = 1mm$?

$$I = \vec{J}\cdot\vec{A} = 10^6\,\frac{C}{s}\cdot\frac{1}{m^2}\cdot\pi\,(10^{-6}\,m^2) = \pi\ \text{Amps.}$$

What are the units of capacitance? Farads.

What are Farads in other units?

$C = \frac{Q}{\Delta V}$ ← coulombs, ← volts.

$1\,F = 1\,\frac{\text{Coulombs}}{\text{Volt}} = \frac{1\,\text{Coulomb}}{1\,J/C} = 1\,\frac{C^2}{J}$

What are the units of resistance? Ohms (Ω)

What is an ohm in other units.

$V = IR$

$R = \frac{V}{I} = \frac{J/C}{C/s} = J \cdot s$

$1\,\Omega = \boxed{1\,J\,s}$

What are the units of E?

$\frac{N}{C}$ or $\frac{V}{m}$

If $\vec{E} = 1\,V/m\,\hat{i}$, what is the voltage difference between $x = 1\,m$ and $x = 5\,m$?

$\Delta V = 4$ Volts.