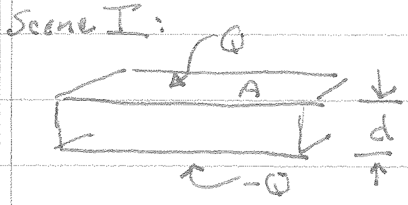
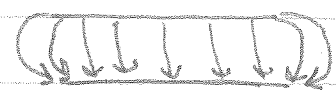
Flashcards: Part 1

Scene I:



A parallel plate capacitor is charged. The spacer is a dielectric with dielectric constant K.

Sketch the electric field lines.



How large is the electric field between the plates? Above and below the plates? Inside the metal plates?

$$E = \frac{\sigma}{\varepsilon} = \frac{Q}{A\varepsilon} = \boxed{\frac{Q}{AK\varepsilon_0}}$$

E = 0 above and below

E = 0 inside the metal.

What is the voltage difference between the top plate and the bottom plate?

$$\Delta V = -\int_{bottom}^{top} \vec{E}\cdot d\vec{l}$$

$$= \frac{Q}{AK\varepsilon_0}\cdot d$$

What is the capacitance?

$$C = \frac{Q}{\Delta V} = \frac{Q}{\left(\frac{Q}{AK\varepsilon_0}\, d\right)} = \frac{\varepsilon A}{d}$$

If $\Delta V = 5V$
and $Q = 2$ Coulombs
what is C?

$C = \frac{Q}{\Delta V} = ~~\frac{5V}{2\,\text{Coulombs}}~~ = ~~2/5\,F~~$

$\frac{2\text{ Coulombs}}{5\text{ Volts}}$

$0.4\,F$

If $C = 1\mu F$
and $\varepsilon = 3\varepsilon_0$
and $A = 0.01\,m^2$,
what is d?

$$d = \frac{3\varepsilon_0 A}{C}$$

$$= \frac{3 \cdot 4\pi\varepsilon_0\,A}{4\pi\,C}$$

$$= \frac{3}{4\pi} \cdot \frac{1}{9\times10^9} \cdot \frac{1}{10^{-6}}\,(0.01)$$

$$= 0.26\,\mu m$$

If $C = 1\mu F$
and $Q = 1\mu C$,
what is the energy stored in the electric field?

$$U = \frac{Q^2}{2C} = \frac{(1\mu C)^2}{2\times 1\mu F}$$

$$= 0.5\,\mu J$$

If $K = 5$, what is the electric susceptibility?

$$\varepsilon = \varepsilon_0 + \chi$$

$$= \varepsilon_0\left(1 + \frac{\chi}{\varepsilon_0}\right)$$

$$= \varepsilon_0 \cdot K$$

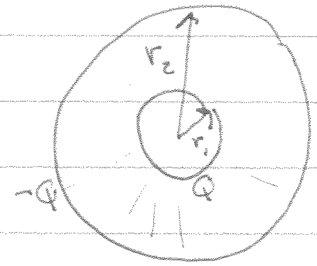
$$\rightarrow K = 1 + \chi/\varepsilon_0$$

$$\therefore \chi = \varepsilon_0(K-1) = \boxed{4\varepsilon_0}$$

For a given charge Q, will E be larger or smaller when the dielectric has a large dielectric constant?

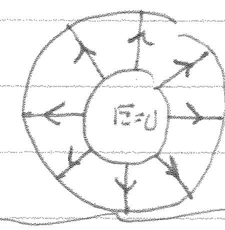
E will be <u>smaller</u>; Instead of $\frac{\sigma}{\varepsilon_0}$, it will be $\frac{\sigma}{\varepsilon}$, where $\varepsilon = K\varepsilon_0$ and $K > 1$.

Scene II



A spherical capacitor is made of two concentric spherical shells.

Sketch the lines of electric field.



For a dielectric with permitivity that is three times larger than the permittivity of free space, what is E(r)?

$$E(r) = \frac{Q}{4\pi \varepsilon r^2}$$

with $\varepsilon = 3\varepsilon_0$.

What is the voltage difference between the inner shell and the outer shell?

$$\Delta V = -\int_{outer}^{inner} \vec{E}\cdot d\vec{l}$$

$$= -\frac{Q}{4\pi\varepsilon}\int_{r_2}^{r_1} \frac{dr}{r^2}$$

$$= \frac{Q}{4\pi\varepsilon}\left.\frac{1}{r}\right|_{r_2}^{r_1}$$

$$= \frac{Q}{4\pi\varepsilon}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

What is the capacitance?

$$C = \frac{Q}{\Delta V} = \frac{Q}{\left(\frac{Q}{4\pi\varepsilon}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\right)}$$

$$= \frac{4\pi\varepsilon}{\frac{1}{r_1} - \frac{1}{r_2}}$$

What is the capacitance of an isolated sphere?

set $r_2 \to \infty$

$$C = \frac{4\pi\varepsilon}{\frac{1}{r_1} - \frac{1}{\infty}} = 4\pi\varepsilon r_1$$

($\frac{1}{\infty} \to$ zero)

If an isolated sphere in free space is charged to a voltage of 100,000 Volts (relative to V = 0 at infinity), what is Q if the radius of the sphere is 15cm?

$$C = \frac{Q}{\Delta V}$$

$$Q = C\Delta V = 4\pi\varepsilon_0 r \Delta V$$

$$= \frac{1}{9\times10^9} \times (0.15) \times 1\times10^5$$

$$= 1.67\ \mu\ \text{Coulomb}.$$

What is E(r = 15cm) just outside the metal surface?

$$E = \frac{\sigma}{\varepsilon_0} = \frac{1.67\times10^{-6}\ \text{Coulomb}}{4\pi(.15)^2 \cdot \varepsilon_0}$$

$$= \frac{(9\times10^9)(1.67\times10^{-6})}{(.15)^2} = 6.7\times10^5\ \frac{V}{m}$$

What is the threshold for dielectric breakdown of dry air?

$E_{max} \cong 3\times10^6$ V/m.

(6.7×10^5 V/m will be possible).