Formula Sheet
PHYC/ECE464

Hermite-Gaussian Beams

$$\frac{E(x,y,z)}{E_0} = H_m\left(\frac{\sqrt{2}x}{w(z)}\right)H_p\left(\frac{\sqrt{2}y}{w(z)}\right)e^{\left(-\frac{x^2+y^2}{w(z)^2}\right)}\frac{w_0}{w(z)}e^{\left(-i\frac{kr^2}{2q(z)}\right)} \times exp\left(-i\left[kz-(1+m+p)tan^{-1}\left(\frac{z}{z_0}\right)\right]\right),$$

Low-order HG modes

$$\begin{cases} H_0(u) = 1;\, H_1(u) = 2u \\ H_2(u) = 2(2u^2-1) \end{cases}$$

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda_0}{\pi n w^2(z)}, \quad w^2(z) = w_0^2\left(1+\frac{z^2}{z_0^2}\right), \quad R(z) = z\left(1+\frac{z_0^2}{z^2}\right), \quad z_0 = \frac{\pi n w_0^2}{\lambda_0}, \quad k = n\frac{\omega}{c} = \frac{2\pi n}{\lambda_0},$$

Divergence

$$\theta_0 = \frac{2\lambda}{\pi\omega_0}$$

Optics

Irradiance $I =< S >= \frac{nc\varepsilon_0}{2}E_0^2$

Fresnel coefficients:

$$r_{\parallel} = \frac{n_t cos\,(\theta_i) - n_i cos(\theta_t)}{n_t cos(\theta_i) + n_i cos(\theta_t)} = \frac{\tan(\theta_i-\theta_t)}{\tan(\theta_i+\theta_t)}$$

$$r_{\perp} = \frac{n_i cos(\theta_i) - n_t cos(\theta_t)}{n_i cos(\theta_i) + n_t cos(\theta_t)} = \frac{\sin(\theta_i-\theta_t)}{\sin(\theta_i+\theta_t)}$$

Intensity reflectivity: $R = |r|^2$

Snell Law: $n_1 sin(\theta_1) = n_2 sin(\theta_2)$

Brewster and critical angles 1->2:

$$\tan(\theta_B) = (n_2/n_1)$$

$$\sin(\theta_c) = (n_1/n_2)$$

n complex: $n \rightarrow \tilde{n} = n + i\kappa$

Lens transformation Gaussian beam:	Lens-maker s' formula:
$\frac{1}{R_2} = \frac{1}{R_1} - \frac{1}{f}$	$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Fabry-Perot Transmission and Reflection (with gain/loss)

$$T(\theta, G_0) = \frac{G_0(1-R_1)(1-R_2)}{\left(1-G_0\sqrt{R_1R_2}\right)^2 + 4G_0\sqrt{R_1R_2}sin^2(\theta)}$$

$$R(\theta, G_0) = \frac{\left(\sqrt{R_1}-\sqrt{R_2}\right)^2 + 4G_0\sqrt{R_1R_2}sin^2(\theta)}{\left(1-G_0\sqrt{R_1R_2}\right)^2 + 4G_0\sqrt{R_1R_2}sin^2(\theta)}$$

$$2\Delta\theta_{1/2} = \frac{1-G_0\sqrt{R_1R_2}}{G_0^{1/2}\sqrt[4]{R_1R_2}}$$

$$\theta = kd = \frac{\omega n d}{c}$$

Finesse:

$$F = \frac{\pi\sqrt[4]{R_1R_2}}{1-\sqrt{R_1R_2}} = \frac{\Delta\upsilon_{FSR}}{\Delta\upsilon_{1/2}}$$

FSR: $\Delta\upsilon_{FSR} = \frac{c}{2nd} = \frac{1}{\tau_{RT}}$

Photon: $\tau = \frac{\tau_{RT}}{1-R_1R_2} \approx \frac{1}{2\pi\Delta\upsilon_{1/2}}$

Resonance condition: $RTPC = q2\pi$

ABCD matrices $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$, det(.)=1; $\begin{pmatrix} r_2 \\ r_2' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} r_1 \\ r_1' \end{pmatrix}$,

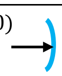
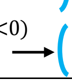
Cavity s-round trip

$r_s = r_{max}\sin(s\theta + \alpha)$

$-1 < cos\theta = (A+D)/2 < 1$

q *parameter at a cavity*

$$\frac{1}{q} = -\frac{A-D}{2B} - i\frac{\sqrt{1-\left(\frac{A+D}{2}\right)^2}}{B}$$

Free Space of length d $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$	Dielectric interface 1->2 $\begin{pmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{pmatrix}$	ABCD law for Gaussian beams $q_2 = \frac{Aq_1+B}{Cq_1+D}$, $q(z) = z + iz_0$ Stability: $-1 < cos\theta = (A+D)/2 < 1$;
Medium of length d, index n immersed in vacuum n_1=1 $\begin{pmatrix} 1 & d/n \\ 0 & 1 \end{pmatrix}$	Thin lens $\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$	Gaussian Pulse propagation $ng = n + v\frac{dn}{dv}$; $Vg = c/\,ng$ $\tau_p^2(z) = \tau_{p0}^2\left(1+\frac{z^2}{L_{GVD}^2}\right)$, $L_{GVD} = \frac{\tau_{p0}^2}{\lvert\beta_2\rvert}$ GVD: $\beta_2 = \frac{\lambda^3}{2\pi c^2}\frac{d^2n}{d\lambda^2} = \frac{\lambda^2}{2\pi c}D$
Curved mirror (R) $\begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix}$ (R>0)  (R<0) 	Spherical dielectric interface $\begin{pmatrix} 1 & 0 \\ (1-n_1/n_2)/R & n_1/n_2 \end{pmatrix}$	Photon density $\frac{N_p}{V} = \frac{I}{h\upsilon c/n_g}$

Gain in a two-level system: $\gamma(\upsilon) = \sigma(\upsilon)\left[N_2 - \frac{g_2}{g_1}N_1\right]$. Gain cross section: $\sigma(\upsilon) = A_{21}\frac{\lambda^2}{8\pi n^2}g(\upsilon)$ Lineshape normalization $\int g(\upsilon)\,d\upsilon = 1$ Beer's law: $\frac{1}{I}\frac{dI}{dz} = -\alpha(I) + \gamma(I)$ Gain/ absorption saturation, homogenously- broadened system: $\gamma(I) = \frac{\gamma_0}{1+I/I_S}$ $\alpha(I) = \frac{\alpha_0}{1+I/I_S}$ $I_S = \frac{h\upsilon}{\sigma(\upsilon)\tau_2}$ Einstein's relations and black body: $\frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h\upsilon^3}{c^3}$, $g_2B_{21} = g_1B_{12}$; $\rho(\nu) = \frac{8\pi n^3\upsilon^2}{c^3}$; $\frac{N_2}{N_1} = \frac{g_2}{g_1}e^{-(E_2-E_1)/kT}$

Lorentzian lineshape $g(\upsilon) = \frac{\Delta\upsilon_h/2\pi}{(\upsilon-\upsilon_0)^2+(\Delta\upsilon_h/2)^2}$ *Normalized Lorentzian* $\tilde{g}(\upsilon) = \frac{g(\upsilon)}{g(\upsilon_0)} = \frac{(\Delta\upsilon_h/2)^2}{(\upsilon-\upsilon_0)^2+(\Delta\upsilon_h/2)^2}$	*Doppler broadened lineshape* $g(\upsilon) = \left(\frac{4ln2}{\pi}\right)^{1/2}\frac{1}{\Delta\upsilon_D}exp\left[(-4ln2)\left(\frac{\upsilon-\upsilon_0}{\Delta\upsilon_D}\right)^2\right]$ $\Delta\upsilon_D = \left(\frac{8kTln2}{Mc^2}\right)^{1/2}\upsilon_0$ *Saturated amplifier* $\ln(G) + \frac{I_1}{I_s}(G-1) = \gamma_0(\upsilon)l_g$

$\frac{dN_P}{dt} = \frac{G^2S-1}{\tau_{RT}}N_P + N_2c\sigma$ (Photon number dynamics due to stimulated and spontaneous emission)

S (survival factor)=R_1R_2 for a simple two mirror linear cavity; G^2=r.t. gain ($\exp(2\gamma L_g)$)
Threshold condition: $SG^2=1$ (linear cavity), SG=1 (ring cavity)

Schawlow-Townes limit for laser linewidth: $\Delta\upsilon_{OSC} \approx 2\pi\frac{h\upsilon}{P_{OUT}}\left(\Delta\upsilon_{1/2}\right)^2$
At steady-state: $\gamma = \gamma_{th} = \frac{\gamma_0}{1+I/I_S}$ (for homogenously broadened)

Inside the gain medium: $I \approx I^+ + I^- \approx 2I^+$ for a high-Q linear (standing-wave) or bidirectional ring cavity, $I \approx I^+$ for a unidirectional ring cavity.
$I_{out} = T_a..T_2I^+$ (T_2 is the output coupling transmission and T_a… are the transmission of other optical surfaces in the path)
Self consistency in ring laser provides intensity I_2 at the end of the gain medium $I_2 = I_s\frac{\gamma_0 l_g - \ln(1/S)}{1-S}$

Q-Switching and Gain-Switching: $\Delta t_p \approx \tau_p$ (cavity photon lifetime)
Power output $P_{out} = h\upsilon\frac{N_p}{\tau_p}\eta_{cpl}$ with η_{cpl} the coupling efficiency
With Photon number $N_p(f) = \frac{1}{2}(n_{th}\ln\left({}^{n_f}/_{n_i}\right) + (n_i - n_f))$ with n_i the initial inversion of atoms.

Modelocking: Repetition Rate $=1/T_{rt}=2Ln_g/c$ (linear cavity), Pulsewidth: : $\Delta t_p \approx> 1/\Delta\nu$

Threshold current density in a diode laser: $J_{th}= eN_{eh}^{th}d/\tau_r$

Physical Constants

$c=2.998\times10^8$ m·s^{-1}	$h=6.63\times10^{-34}$ J·s	$e=1.6\times10^{-19}$ C
$k_B=1.380\times10^{-23}$ J·K^{-1}	$\varepsilon_0=8.85\times10^{-12}$ F/m	$m_e=9.1\times10^{-31}$ kg

1G = 10^{-4} T , 1 eV = 1.602 x 10^{-19} J, 1 dyne = 10^{-5} N, 1 erg = 10^{-7} J