

The Ratio e/m for Electrons

1 Objective

Study the effects of electric and magnetic fields on a charged particle and measure the charge-to-mass ratio (e/m) of the electron.

- Review basic electrodynamics and forces.
- Study and understand Helmholtz coils and production of electrons from an e-gun.
- Study dynamics of a charged particle in an electromagnetic (E and B) field and circular motion.
- Obtain the charge-to-mass ratio (e/m) of the electron from different methods.

2 Theory

In 1897 J.J. Thomson first measured the charge-to-mass ratio (e/m) of the electron. His experiment involved analyzing the electron's motion in an electric and magnetic field. Some time later R.A. Millikan determined the charge of the electron and resolved this ratio using an electric field alone. It was Thomson's hypothesis of the electron's existence, and its charge-to-mass ratio, that led to the concept of the first atomic particle.

Helmholtz coils give us a simple solution for the value of the magnetic field along the axis of symmetry. For coils of N wires each carrying current I , having radius R show that the field along the symmetry axis x is given by,

$$B = \frac{\mu R^2 N I}{(R^2 + x^2)^{\frac{3}{2}}}. \quad (1)$$

In the Helmholtz configuration, $x = R/2$, The permeability of free space $\mu = 4\pi \times 10^{-7} \frac{\text{weber}}{\text{amp-meter}}$. In our apparatus $N = 130$ and $R = .15$ meters, therefore

$$B = (7.8 \times 10^{-4} \frac{\text{weber}}{\text{amp-meter}^2}) \times I. \quad (2)$$

By measuring V,I, and the radius of curvature r we can determine the ratio e/m .

3 Procedure

Fig. 1 shows the apparatus. The e/m tube is filled with helium at a pressure of $\sim 10^{-2}$ mm Hg. Connect a regulated 6-9 Vdc supply rated at 2 A to the Helmholtz coil jacks. Be sure to observe the color-coded polarity on all input jacks. You will need to connect an ammeter in series between the supply and the coil jacks so that you can measure the current to 1% or better accuracy. A wire wound potentiometer control to the right of the coil jacks on the panel serves to adjust the current in the coils.

Now connect a 6.3 V (ac or dc, but do *not* exceed 6.3 V!) supply rated at 1.5 A to the heater jacks of the electron gun at the right side of the base panel. Besides powering the heating element, this supply also powers the small scale lamps on the coil brackets.

Lastly connect a high voltage source of 150-300 Vdc rated at 40 mA to the Electrode jacks of the electron gun. The high voltage applied to the anode of the gun accelerates the electrons boiled off from the cathode to form the necessary electron beam. The value of this voltage determines the average velocity of the electrons in the beam and appears in the final expression for the ratio of e/m . Regardless of the metering on your high voltage supply, the accelerating voltage must be measured with an accurate dc voltmeter connected at the jacks labelled Voltmeter on the base panel.

Key to Figure 1	
E	Envelope to vacuum tube
G	Electron Gun
F	Focusing element of gun
Z	Heater or filament of gun within cathode
A	Anode of gun
S	Electron Beam
H	Helmholtz Coils
B	Space of magnetic field
M	Mirror scale
W	Lamps to measure r
All Others	Various connectors as labelled

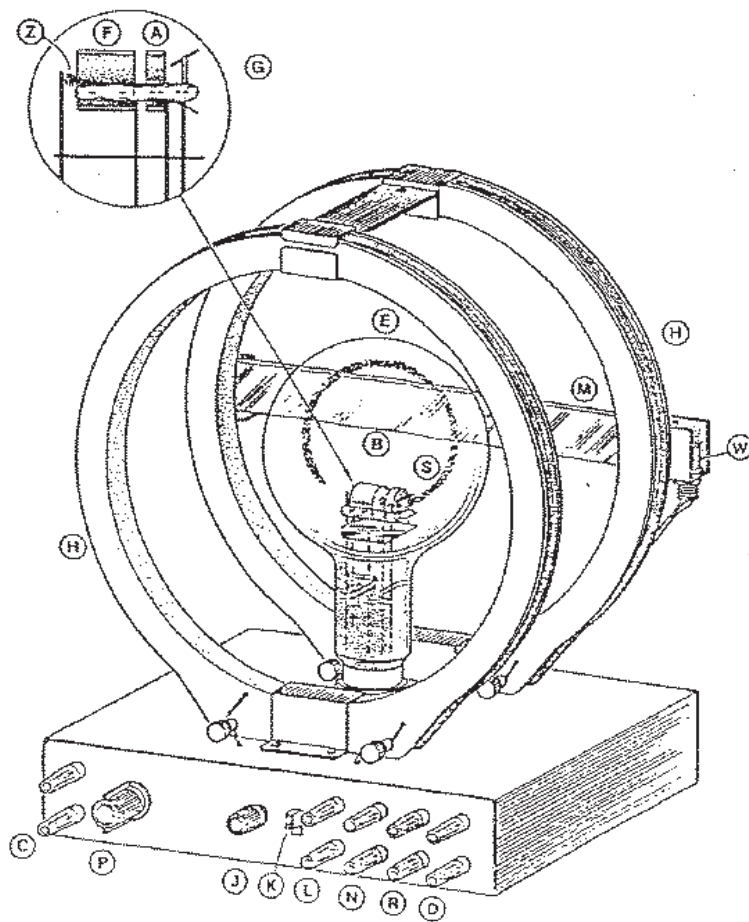


Figure 1: Charge-to-Mass Ratio Apparatus.

Turn the *Current Adjust* control to zero and set the switch on the panel to the e/m position. Nothing should be connected to the jacks labelled *Deflection Plates* at this time. Turn on the *Heater* supply and allow the electron gun filament to heat up for two minutes. At the end of this time apply a 200 Vdc potential from the high voltage supply to the *Electrodes* for the initial value of accelerating voltage.

With the accelerating voltage on, you should be able to see the violet hue of the electron beam projecting horizontally from the electron gun. For proper viewing of the beam this experiment should be conducted in a darkened room or in subdued lighting using a black cloth hood to mask the tube and to backdrop the beam. Turn on the coil current and increase the *Current Adjust* control from zero to a current value that bends the beam around into a complete circle on itself. It may be necessary to rotate the tube socket until the end of the curving beam strikes between the two wire leads supplying the heating filament on the left side of the electron gun. This brings the beam and gun into an exact parallel position with the plane of the coils.

4 Data

Three data variables are necessary to make a determination of the ratio e/m . They are the accelerating voltage, the coil current, and the radius of the beam curvature. Each of these variables must be measured as accurately as is possible in order that e/m come close to the accepted value of $1.76 \times 10^{11} \frac{\text{coul.}}{\text{kg}}$. However, the electron beam radius involves by far the largest error because the beam is enclosed in a glass envelope forcing a sighting measurement to be made at a distance which is fraught with parallax error. To overcome this defect, an anti-parallax scale is mounted and illuminated by two small lamps on the back of the coil. The technique for measuring the beam radius is to move your head in order to align the electron beam with the reflection of the beam that you see on the mirrored scale. Each side of the beam loop may have a slightly different radius of curvature. Therefore, take a mean of the two radii for the value of r .

Data collection and calculations:

- **e/m from Scattered plots:** For about ten different combinations of accelerating voltage V and coil current I record the corresponding value of the radius r . Calculate e/m and its error.
- **e/m as a function of I for constant V :** Collect data for the radii by varying the current I for a constant voltage. Plot r vs. I^{-1} at const. V , and determine e/m from a linear least squares fit (with unequal errors). Take data for a6-10 different values of I and realize at least 4 runs.
- **e/m as a function of V for constant I :** Repeat the procedure above but varying V for

fixed I via plotting r^2 vs. V at const. I .

You now have three results for e/m . Compare the three including their errors with the accepted value.

Note: The greatest source of systematic error in this experiment is the velocity of the electrons, v_e . First, the non-uniformity of the accelerating field caused by the hole in the anode causes v_e to be slightly less than its theoretical value. Second, collisions with the helium atoms in the tube further rob the electrons of their velocity. Since e/m is proportional to $1/r^2$, and r is proportional to v_e , experimental values for e/m will be greatly affected by these two effects. To minimize this error you can try to measure radii to the *outside* of the beam path. To minimize the relative effect of collisions, keep the accelerating voltage as high as possible (above ~ 250 V tends to give best results). On the other hand, if the voltage is too high, the radius measurement will be distorted by the curvature of the glass at the edge of the tube. Best results are typically obtained for radii of less than ~ 5 cm. Your experimental values for e/m *will* be higher than theoretical, due to the fact that both sources of error mentioned cause the radius to be measured as smaller than it should be.

Also go through the qualitative experiments suggested next.

5 Qualitative Experiments

You can qualitatively demonstrate the vector effects of an electric and magnetic field on a charged particle with the e/m apparatus as predicted by the Lorentz force law.

With the electron beam on and the Helmholtz coils operating, rotate the glass envelope and observe the resulting spiral path of the beam in the magnetic field. How does the Lorentz vector expression account for this spiraling? Turn the coil current down to zero and then switch the polarity of the coil leads to reverse the current direction which will in turn reverse the magnetic vector. Turn the coil current on again and observe the deflection of the electrons.

An applied electric field will also deflect the electron beam. A pair of skewed plates at the right side of the electron gun produces an electric field around the emerging electron beam when a voltage is applied. To show the electric field deflection, connect in parallel the voltage leads from the *Electrode* jacks to the jacks labelled *Deflection Plates*. Switch on the accelerating voltage supply and set the central switch to the *Electrical Deflection* position. There should be no current in the coils in this case. Note how the beam deflection varies with the voltage value. If the polarity of the plates is in the same sense as the color-coded jacks, can you tell what sign the charge is in the beam? Turn down the voltage to

zero and reverse the polarity of the *Deflection Plate Jacks*. In which direction is the beam deflected now? Explain. Observe what happens when the beam bends and hits the deflection plates.

In retrospect, J.J. Thomson originally determined the charge to mass ratio of the electron by first deflecting an electron beam with an electric field, and then compensating this deflection with a opposing magnetic field deflection. You can reenact this procedure here qualitatively. First deflect the electron beam using the electrical deflection plates (reverse the polarity sense of the jacks). Turn on the current to the Helmholtz coils and adjust the current until the deflection of the magnetic field cancels the deflection of the electric field. This is the substance of the historic Thomson experiment which we cannot reproduce here in a quantitative way due to the uncertainty of the electric field value between the deflection plates.

6 Technical aspects

- Starting with the Biot-Savart law, derive the expression for the magnetic field B provided by the pair of coils in the Helmholtz configuration.
- Starting with the Lorentz force and a charged particle in circular motion, together with conservation of energy, derive the expression for e/m as a function of V , I or B , and radius.

7 Questions

1. Why do we see the electron beam at all?
2. We ignored the Earth's magnetic field in our procedure. How much error does this introduce into this experiment? (see <http://www.ngdc.noaa.gov/seg/geomag/>)
3. Suppose that protons were emitted in the vacuum tube instead of electrons. How would this effect the experiment?
4. Show that if the magnetic field is held constant, the time t required for an electron to make a complete circle in your e/m tube and return to the anode is independent of the accelerating voltage by deriving an expression for this time. The reciprocal $f = \frac{1}{t}$ is called the electron's cyclotron frequency.
5. Would a relativistic correction for the electron's momentum be appreciable for the present experiment?

8 References

- [1] Melissinos and Napolitano, Chapters 1,5,6.
- [3] Data Reduction and Error Analysis for Physical Sciences, 3rd ed. Philip R. Bevington, D. Keith Robinson.