

Equations using MLE method for fitting a straight line to a series of data with uneven errors

Given a series of “ N ” measurements $\{y_i\}$ of a quantity “ y ” of as we vary “ x ” through the points $\{x_i\}$, respectively, and where the data are expected to have the linear form “ $y = mx + b$ ”, we can find the most probable estimates, or best estimates using MLE method, for the slope m (\hat{m}) and the intercept b (\hat{b}) as (Eq. 6.12 Bevington):

$$\hat{m} = \frac{1}{\Delta} \left(\sum \frac{1}{\sigma_i^2} \sum \frac{x_i y_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2} \sum \frac{y_i}{\sigma_i^2} \right)$$

$$\hat{b} = \frac{1}{\Delta} \left(\sum \frac{x_i^2}{\sigma_i^2} \sum \frac{y_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2} \sum \frac{x_i y_i}{\sigma_i^2} \right)$$

$$\Delta = \sum \frac{1}{\sigma_i^2} \sum \frac{x_i^2}{\sigma_i^2} - \left(\sum \frac{x_i}{\sigma_i^2} \right)^2$$

Where “ σ_i ” is the standard deviation of a particular measurement point “ y_i ”. The errors in the estimated parameters \hat{m} and \hat{b} are (Eq. 6.21-22 Bevington)::

$$\sigma_m^2 = \frac{1}{\Delta} \sum \frac{1}{\sigma_i^2}$$

$$\sigma_b^2 = \frac{1}{\Delta} \sum \frac{x_i^2}{\sigma_i^2}$$

This method is described in Bevington Chapter 6 sections 6.1-4.