Homework 3

(Due Date: Monday, Feb. 12)

Problem 1.- Given the function $f(u, v)$ which depends on variables u and v with variances ${\sigma_u}^2$ and ${\sigma_v}^2$, respectively, obtain the variance ${\sigma_f}^2$ of $f(u, v)$ in terms of the variances ${\sigma_u}^2$ and ${\sigma_v}^2$ and the covariance σ_{uv} of u and v for the following cases where a and b are positive constants:

(a) $f = au \pm bv$ (b) $f = \pm auv$ (c) $f = \pm au/v$

(d) $f = au^{\pm b}$ (e) $f = ae^{\pm bu}$ (f) $f = a \ln(\pm bu)$

Find the uncertainty σ_x, in "x" as a function of the uncertainties σ_u, and σ_v, in *u* and *v* for the following functions:

(g) $x = \frac{1}{2(u+v)}$ (h) $x = \frac{1}{2(u-v)}$ (i) $x = \frac{1}{u^2}$ (d) $x = uv^2$ (j) $x = u^2 + v^2$

Note: for cases from (a) to (c) assume that the variables u and v are correlated. For cases from (g) to (j) assume that the variables u and v are not correlated.

Problem 2:
The Doppler shift describes the frequency change when a source of sound waves of frequency "f" moves with a velocity "v" towards an observer at rest as $\Delta f = fv/(u - v)$, where "u" is the velocity of sound. Determine the Doppler shift and its uncertainty for the situation when

$u = (332 \pm 8)$ m/s
$v = (0.123 \pm 0.003)$ m/s
$f = (1000 \pm 1)$ m/s

What is the quantity that contributes the least and the most to the uncertainty of the Doppler shift?

Problem 3.- The period T of a pendulum is related to its length by the relation

$$T = 2\pi\sqrt{\frac{L}{g}},$$

where g is the gravitational acceleration. Suppose you are measuring g from the period and length of the pendulum. You have measured the length of the pendulum to be 1.1325±0.0014 m. You independently measure the period to within an uncertainty of 0.06%, that is $\sigma_T/T = 6\times10^{-4}$. What is the fractional uncertainty in g (σ_g/g), assuming that the uncertainties in L and T are independent and random?

Problem 4.- Niels Bohr showed that the energy (E_n) of the quantum states of a Hydrogen atom are given by:

$$E_n = -2\pi \frac{m\,e^4}{h^2}\frac{1}{n^2}$$

where *m* is the mass of the electron, *e* is charge and *h* is Planck's constant. *n* is the Principle Quantum Number and is an exact number. Suppose the relative error (σ_u/u) in each of the measured quantities *m*, *e* and *h* is:

Quantity	Rel. Error
m	0.001
e	0.002
h	0.0001

What is the relative error in energy of the third quantum state? Assume all the errors are statistically determined standard deviations of the mean.