Poisson Statistics

1 Introduction

This simple experiment will help you gain familiarity with the second most important statistical distribution in physics, the Poisson distribution. It describes the results of experiments where events occur at random, but at a definite average rate (example: radioactive decays). It is of paramount importance in all of atomic and subatomic physics, in particular.

2 Structure

You will use the same setup as the electron rest mass experiment. In the first part of the experiment, no source will be needed as you will simply measure the background count rate in our NaI detector. You will use our multichannel analyzer (MCA) in multichannel scaling mode (MCS) instead of pulse height analysis mode (PHA). In MCS function, the MCA no longer acts as a pulse height selector, but as a multichannel scaler with each channel acting as an independent scaler. At the start of operation, the MCA counts the incident pulse signals (regardless of amplitude) for a certain dwell time, and stores this number in the first channel. It then jumps to the next channel and counts for another dwell time period, after which it jumps to the next channel and so on. In MCS mode, therefore, the channels represent bins in time. Typical dwell times will be in the millisecond range.

Suggested starting values:

High-voltage bias of 1000V for the NaI, minimum gain (both fine and coarse) for the preamp/amp /discriminator, and no radioactive source.

Setup the software with 256 channels (Note: this means the computer will effectively perform this simple counting experiment 256 times for you!) and MCS mode of operation with 1 pass. Make sure the MCS is set to internal with the presets on. And now the important part: adjust the dwell time such that the average count rate per channel is around 1-2 counts. Save the resulting spectrum in an ASCII or text file. Repeat this procedure for two other dwell times such that the average count rate per channel is around 5 counts and around 10 counts, respectively.

- Discuss the source of these background counts (where does this background come from?)

Repeat the experiment for a radioactive source, for example Cs137 adjusting the dwell time to obtain the same average count rates of around 1-2, 5 and 10 counts respectively.

3 Analysis

Plot your three resulting distributions with statistical error bars [1] for the two cases, (a) background counts in the detector and (b) when using a radioactive source. Use MATLAB. Notice the significant asymmetry of the distribution for the lowest average count rate (Poisson at work!). Calculate and standard deviation of the distribution in each case. How closely do your results follow the expected Poisson distribution, i.e. that the standard deviation is equal to the square the mean? Also notice how your highest average count rate case is rather symmetric, i.e. already for an average count rate of around 10 the Poisson distribution is practically indistinguishable from a Gaussian.

Compare your three count rate distributions with the expected Poisson distributions graphically with statistical error bars, and calculate the chi square (χ^2) per degree of freedom (Section 4.4 in [3])(***discuss the goodness of the assumption***; how well does the experiment approach the theory?). For the highest average count rate case, repeat with a Gaussian distribution. Provide an explanation for this.

In general, Poisson (P) and Gaussian (G) distributions are not the same. Evaluate the Gaussian distribution at the same discrete x-values as the ones defined for your Poisson distribution. Next, normalize both distributions and plot the relative difference of the Poisson distribution from your data and the Gaussian distribution, i.e. (Poisson-Gaussian)/Poisson, for the case without a source, only with background counts for the three investigated average count rate per channel. This quantity is a measure of the asymmetry of the Poisson distribution around the mean. Compare this difference using your results with the theoretical prediction for these difference from Ref. [4].

[1] "The Poisson distribution and statistical uncertainties do not apply solely to experiment where counts are recorded in unit time intervals. In any experiment in which data are grouped in bins according to some criterion to form a histogram or frequency plot, the number of events n_i in each individual bin will obey Poisson statistics with a certain $\mu_i = n_i$ and fluctuate with statistical uncertainties" $\sigma^i_{Poisson}$ (see Chapter 3 [3]).

$$\frac{(G-P)}{P} \simeq \frac{\delta - \delta^3/3\mu}{2\mu} \tag{1}$$

with $\delta = n - \mu$ where μ and $n = 0, 1, 2, ...$ are the parameters of the Poisson distribution. Discuss your results.

4 References

[1] Melissinos and Napolitano, Chapter 10.

[2] Multichannel Analyzer manual can be found in:
http://www.spectrumtechniques.com/ucs30.htm

[3] Data Reduction and Error Analysis for Physical Sciences, 3rd ed. Philip R. Bevington, D. Keith Robinson.

[4] L. J. Curtis, Am. J. Phys. **43**, 1101 (1975).