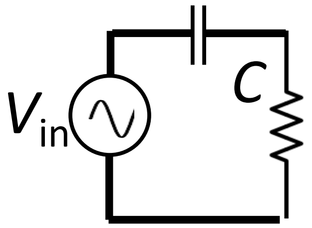
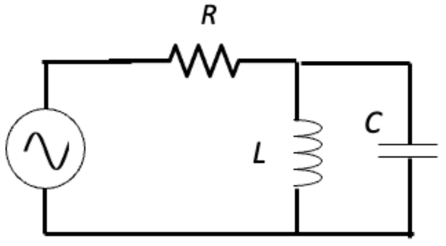
Lab 4: AC circuits (II)

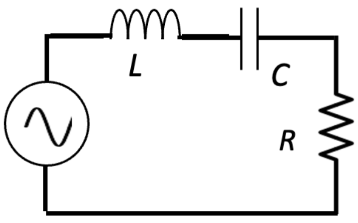
RC high Pass and RLC Resonant Circuits

- Bode Plots
- dB representation in Power and Voltage
- RC High-Pass Filter



- RLC Resonant Circuits
  - Amplitude
  - Phase
  - Resonance and Q-Factor





- *Superposition Theorem*

Remider

AC analysis of circuits using complex numbers

$$z = a + jb = Ae^{j\phi}$$

Amplitude $A = \sqrt{a^2 + b^2}$

Phase $\phi = \tan^{-1}\left(\frac{b}{a}\right)$

Assumptions:

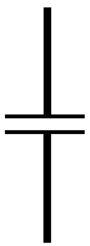
i) Steady-state

ii) Sinusoidal waveforms:

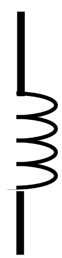
$$V(t) = A\sin(\omega t + \phi)$$

Ohm's Law for L and C:

Impedance (*Z*) Measured in ohms



C $$Z_C = \frac{V_C}{I_C} = \frac{1}{j\omega C}$$

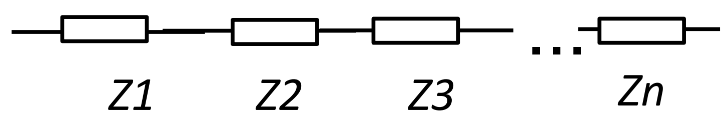


L $$Z_L = \frac{V_L}{I_L} = \frac{\omega L}{-j} = j\omega L$$

R $$Z_R = \frac{V_R}{I_R} = R$$

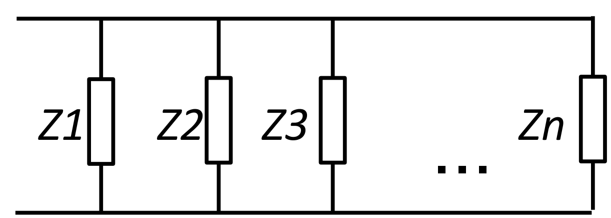
Equivalent Impedances

Series



$$Z_{total} = \sum_{i}^{n} Z_i$$

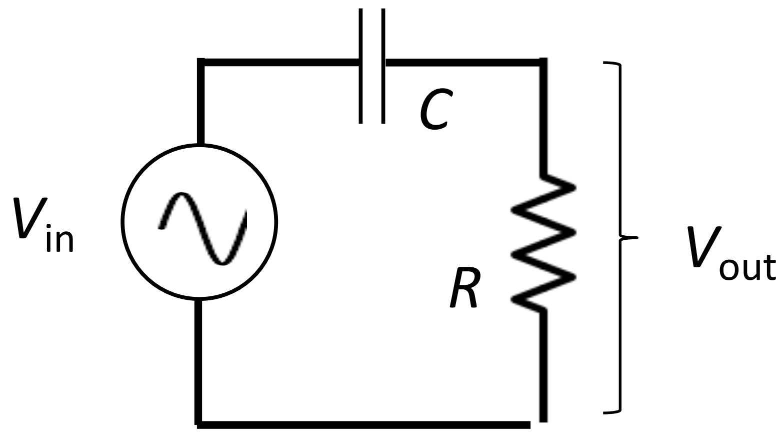
Parallel



$$\frac{1}{Z_{total}} = \sum_{i}^{n} \frac{1}{Z_i}$$

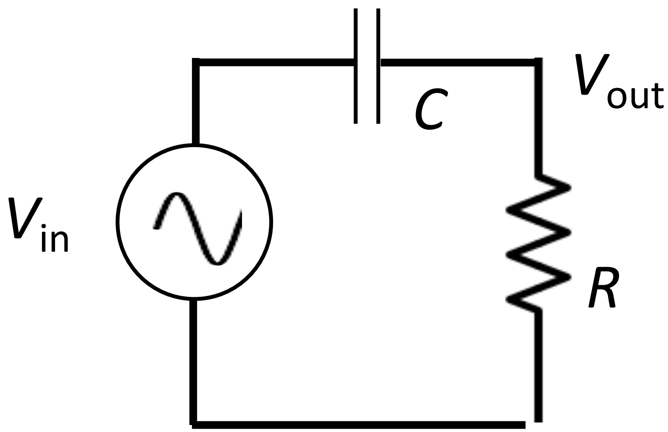
90° phase-shift in polar form: $e^{j\frac{\pi}{2}} = \cos(\frac{\pi}{2}) + j\sin(\frac{\pi}{2}) = j$

AC circuit: High-pass

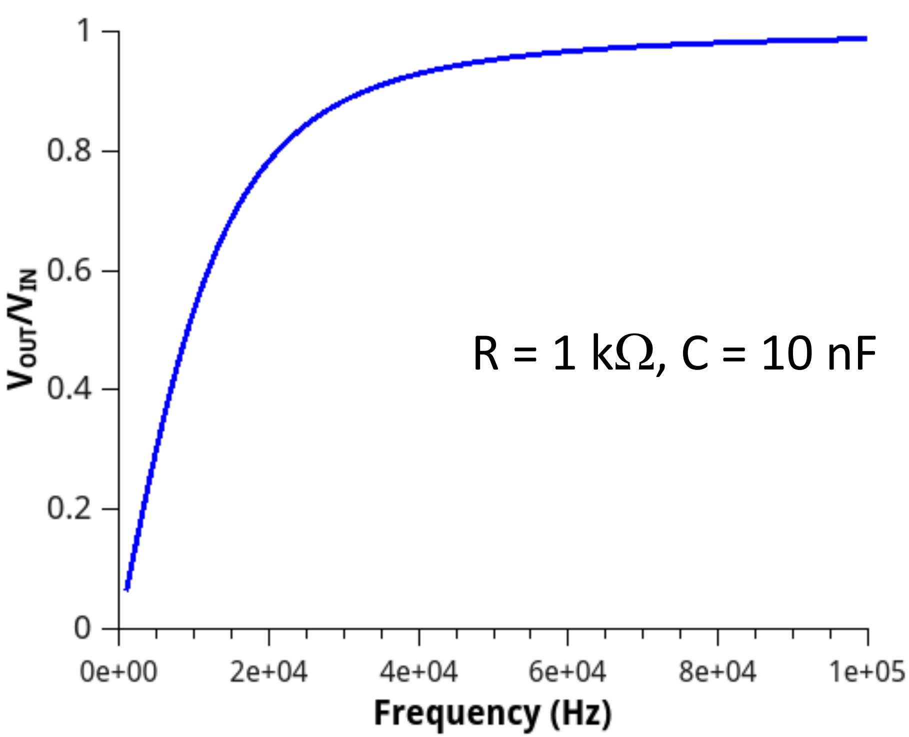


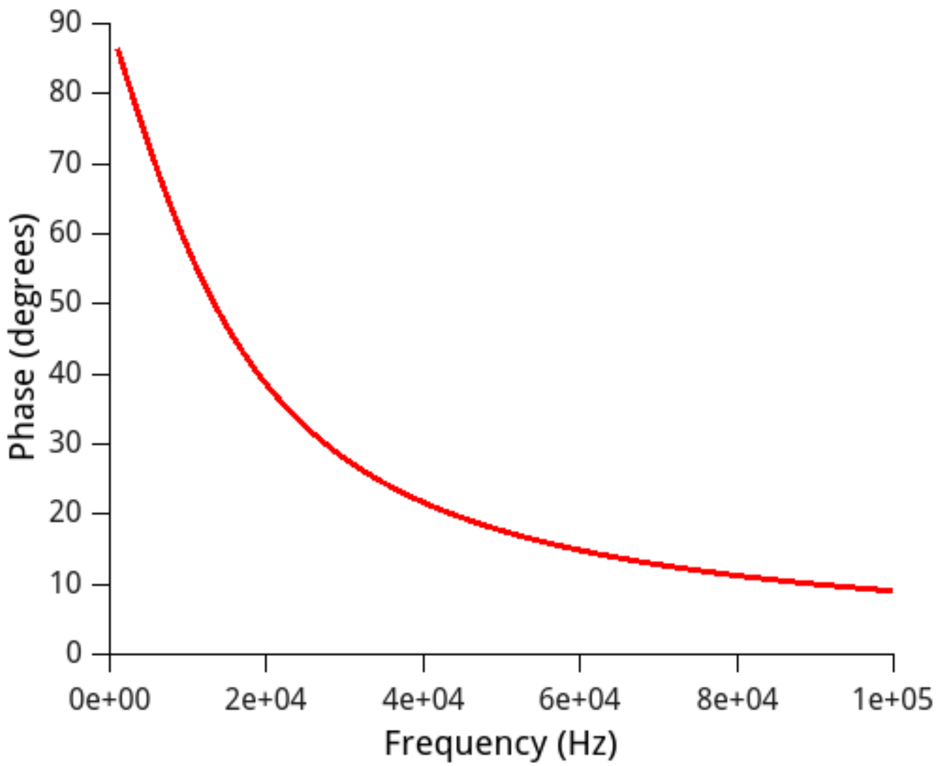
$$\frac{V_{OUT}(\omega)}{V_{IN}(\omega)} = \frac{R}{R + 1/j\omega C}$$

AC circuit: High-pass



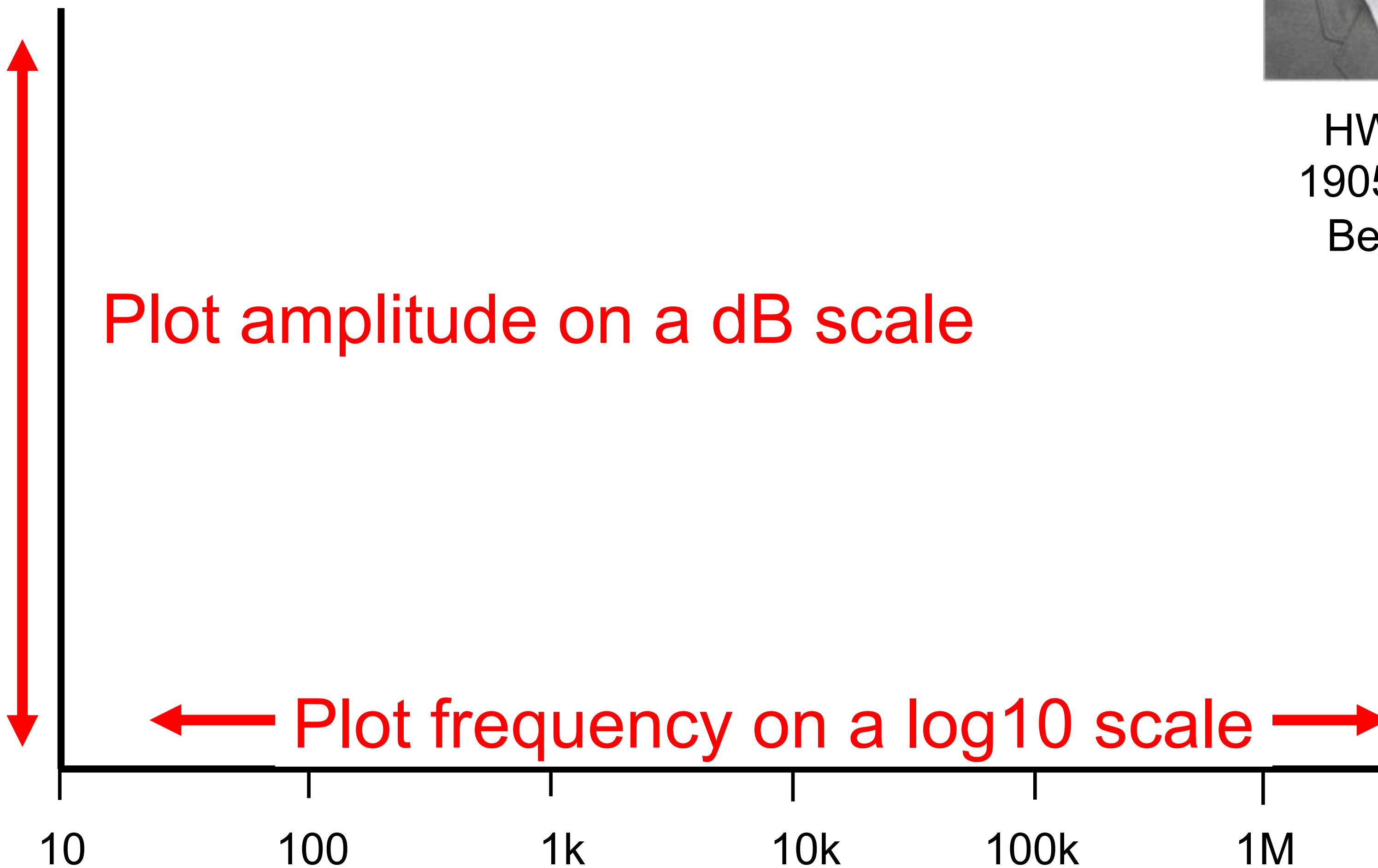
$$\frac{V_{OUT}(\omega)}{V_{IN}(\omega)} = \frac{R}{R + 1/j\omega C}$$





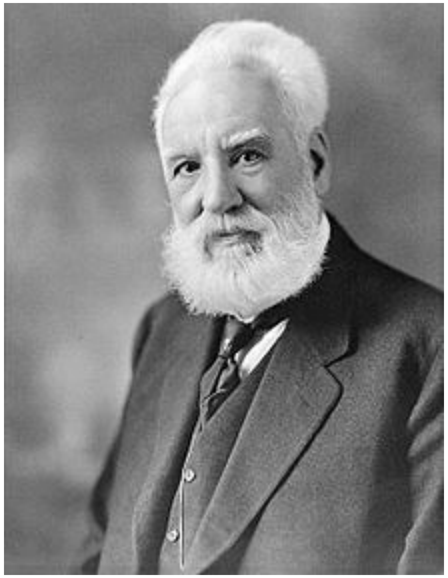
The Bode Plot for $|V_{out}/V_{in}|$

HW Bode
1905—1982
Bell Labs



The Decibel: A Ratio

RATIO	POWER $10 \log_{10} \left\{ \frac{P_{\text{signal}}}{P_{\text{ref}}} \right\}$	FIELD $20 \log_{10} \left\{ \frac{A_{\text{signal}}}{A_{\text{ref}}} \right\}$
1	0 dB	0 dB
10	10 dB	20 dB
100	20 dB	40 dB
2	3 dB	6 dB
0.01	–20 dB	–40 dB

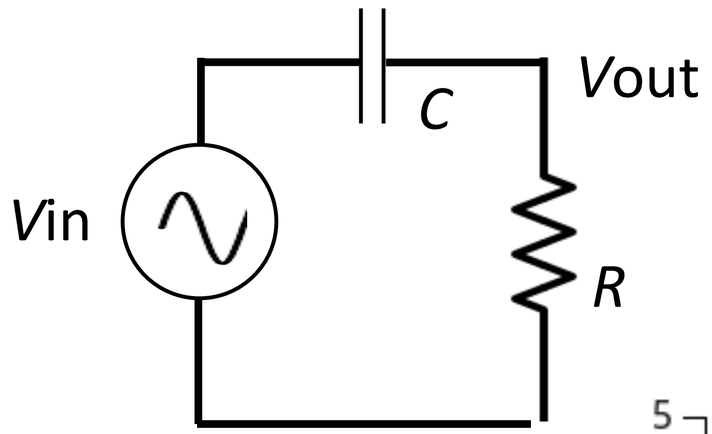


In honor of
Alexander Graham Bell

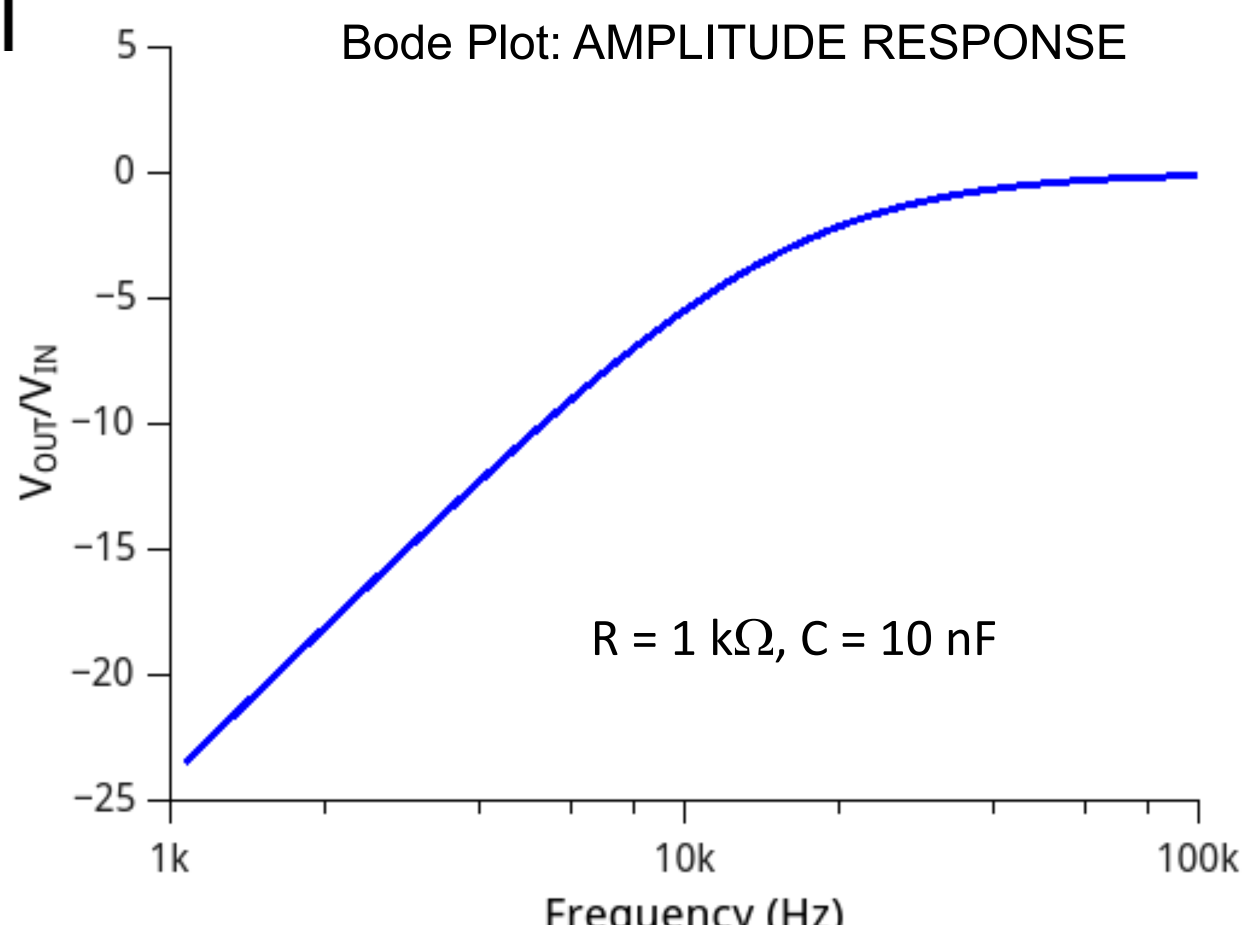
$$\text{dBm: } 10 \log_{10} \left\{ \frac{P_{\text{signal}}}{1 \text{ mW}} \right\}$$

$$0 \text{ dBm} = 1 \text{ mW}$$

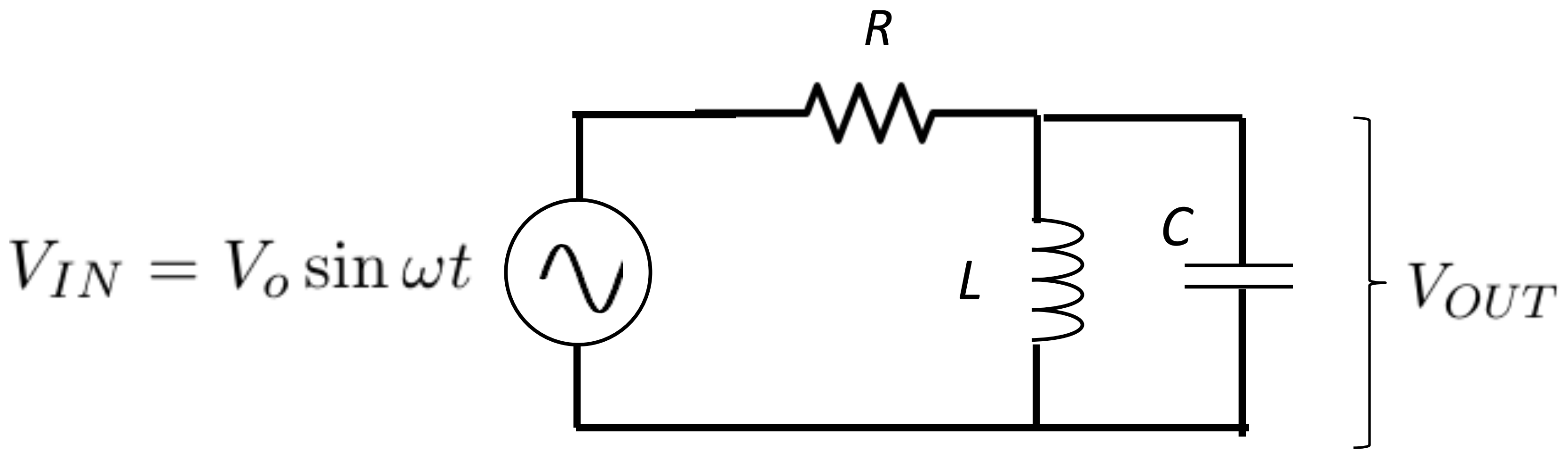
AC circuit: High-pass



$$\frac{V_{OUT}(\omega)}{V_{IN}(\omega)} = \frac{R}{R + 1/j\omega C}$$



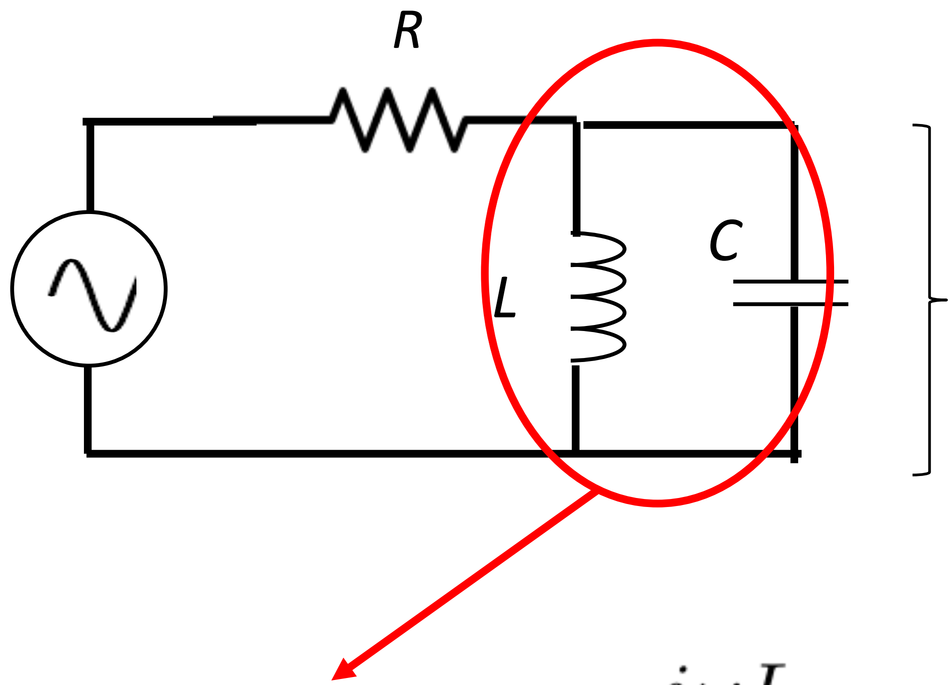
Inductor-capacitor in AC circuit: Resonance



Parallel LC circuit

Inductor-capacitor in AC circuit: Resonance

$$V_{IN} = V_o \sin \omega t$$



$$V_{OUT}$$

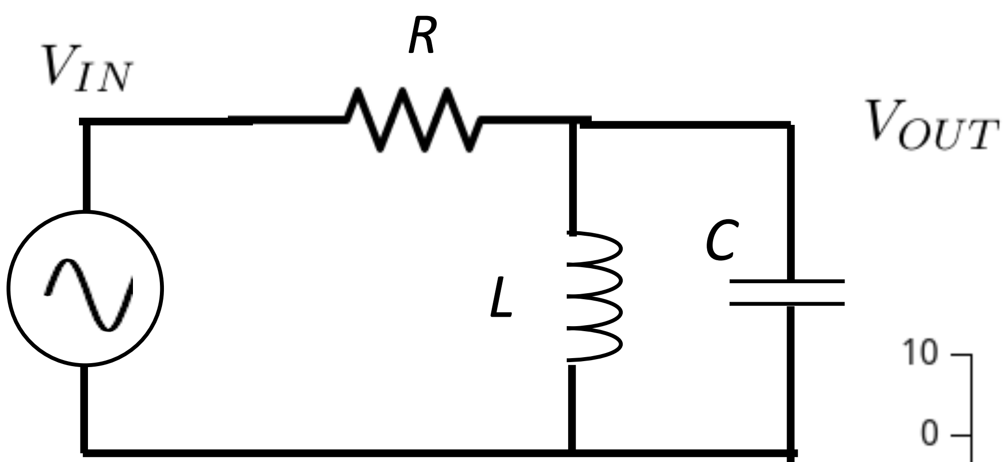
$$Z_{parallel} = \frac{j\omega L}{1 - \omega^2 LC}$$

$$G(\omega) = \frac{V_{out}}{V_{in}} = \frac{Z_p}{Z_p + Z_R}$$

Resonance at:

$$f = \frac{1}{2\pi\sqrt{LC}}$$

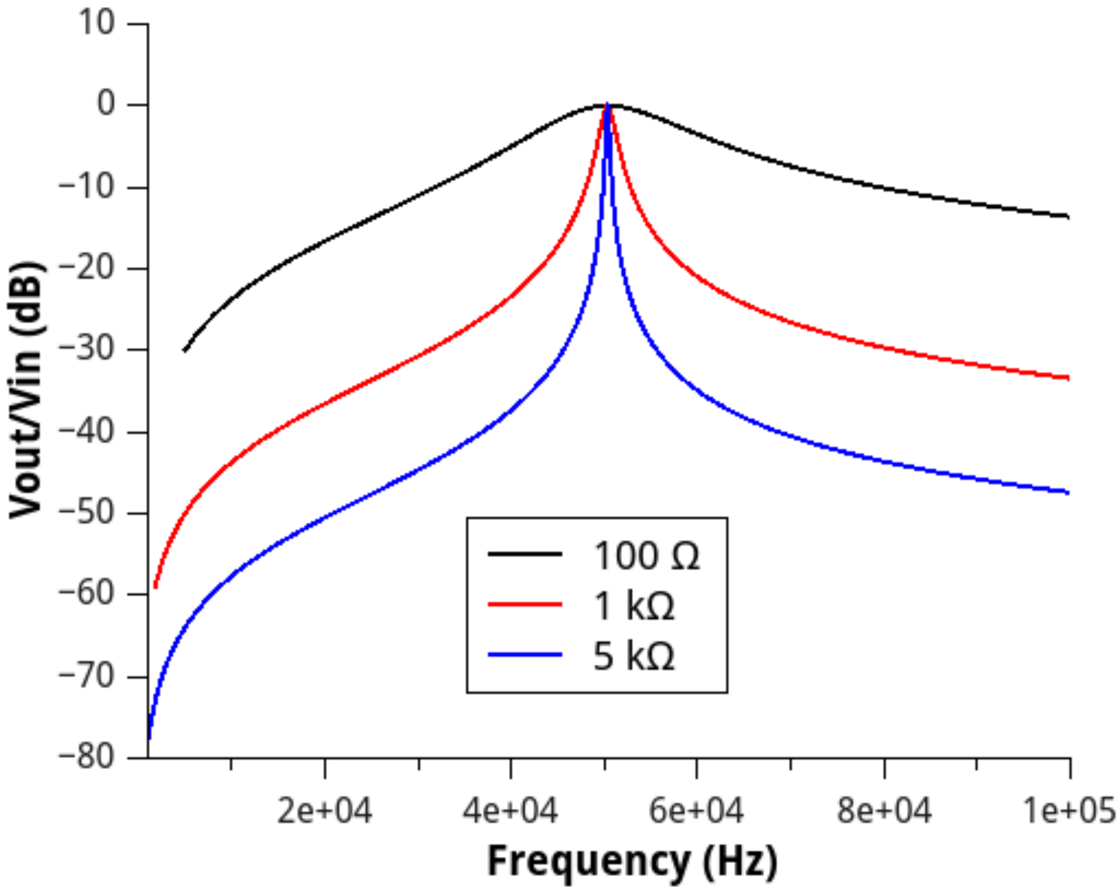
Inductor-capacitor in AC circuit: Resonance



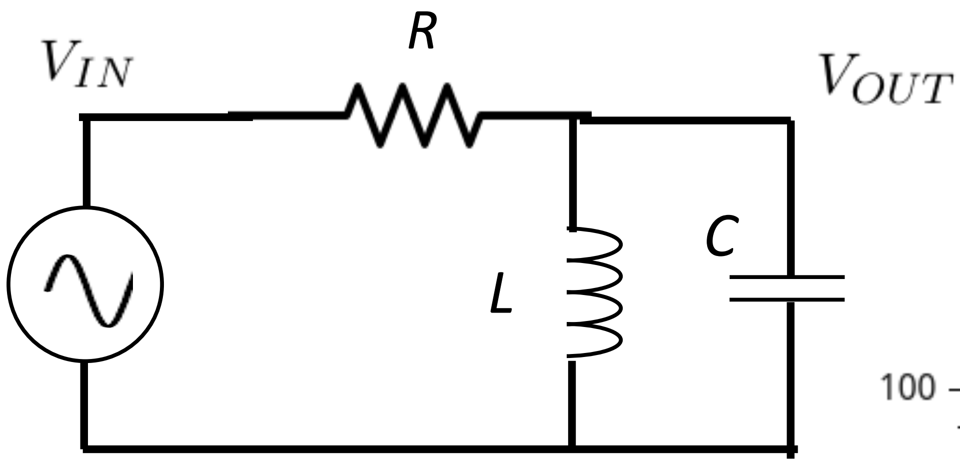
C = 100 nF
L = 100 uH
*f*res = 50.3 kHz

R = 100Ω, 1 kΩ, 5kΩ

AMPLITUDE RESPONSE



Inductor-capacitor in AC circuit: Resonance



C = 100 nF
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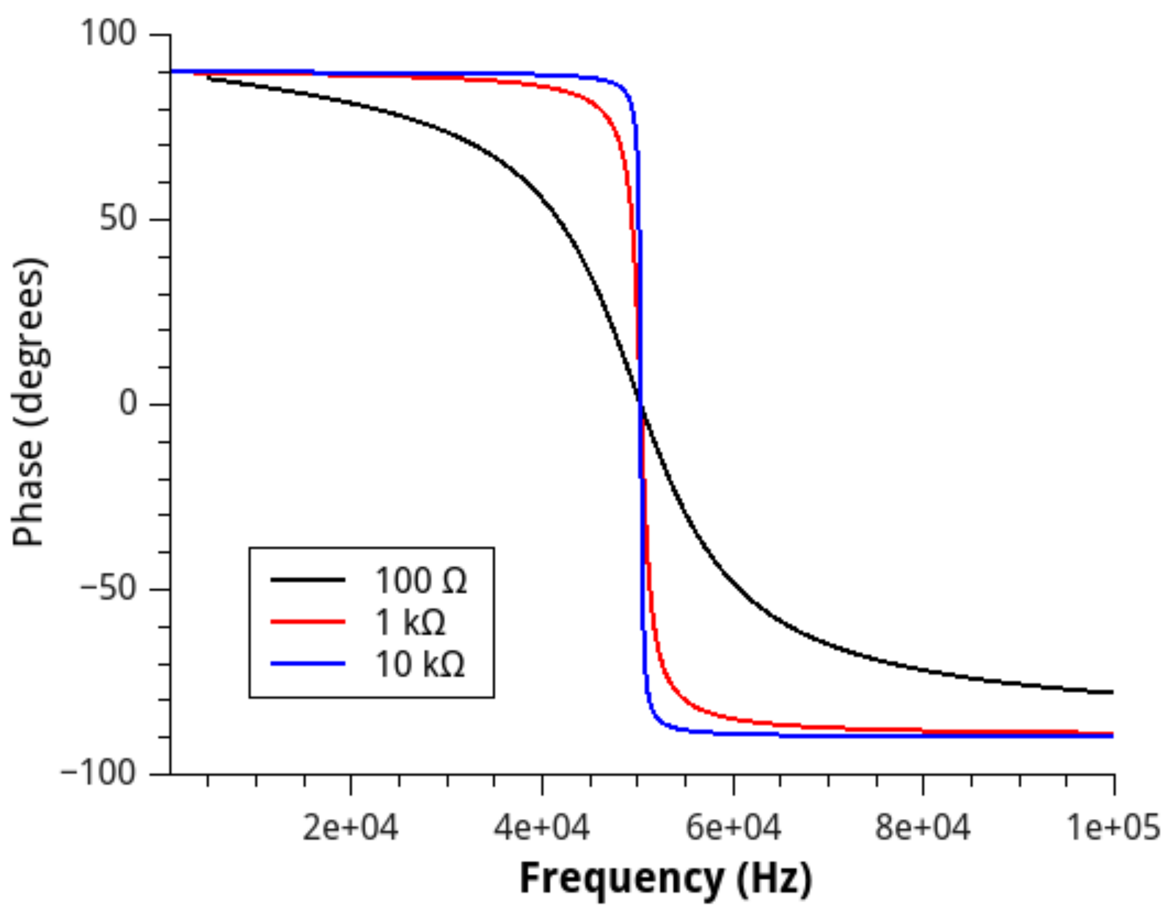
PHASE RESPONSE



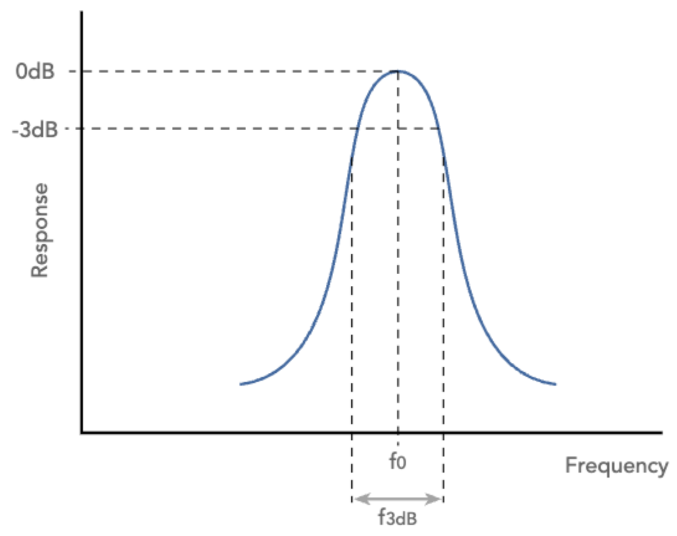
Q-factor: Sharpness of Resonance

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{f_0}{\Delta f}$$

Sharper resonance ⟶ Higher Q

Δf = frequency range between the – 3 dB points

– 3 db ≈ 0.707 of the peak



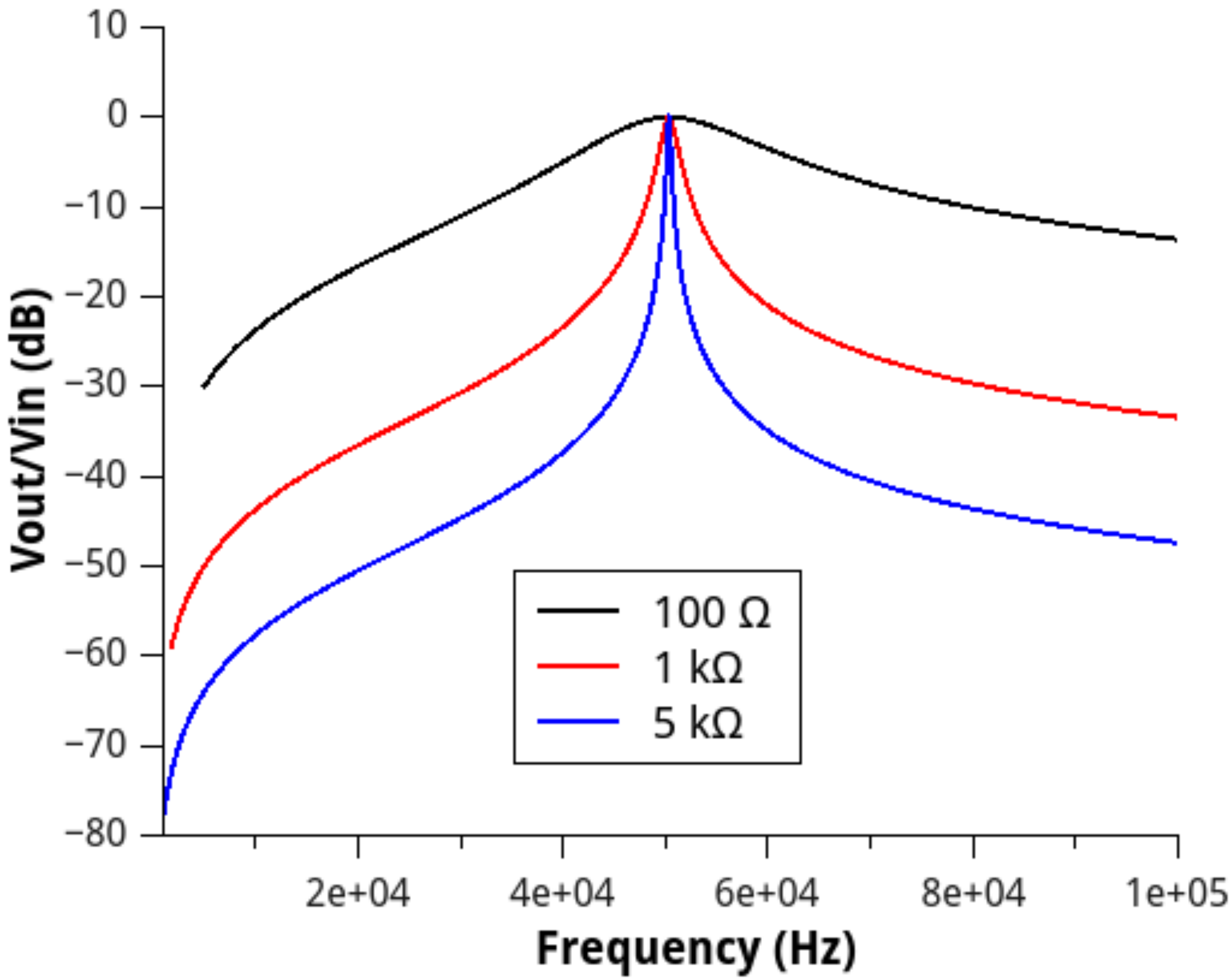
Q-factor: Sharpness of Resonance

Example RLC circuit:

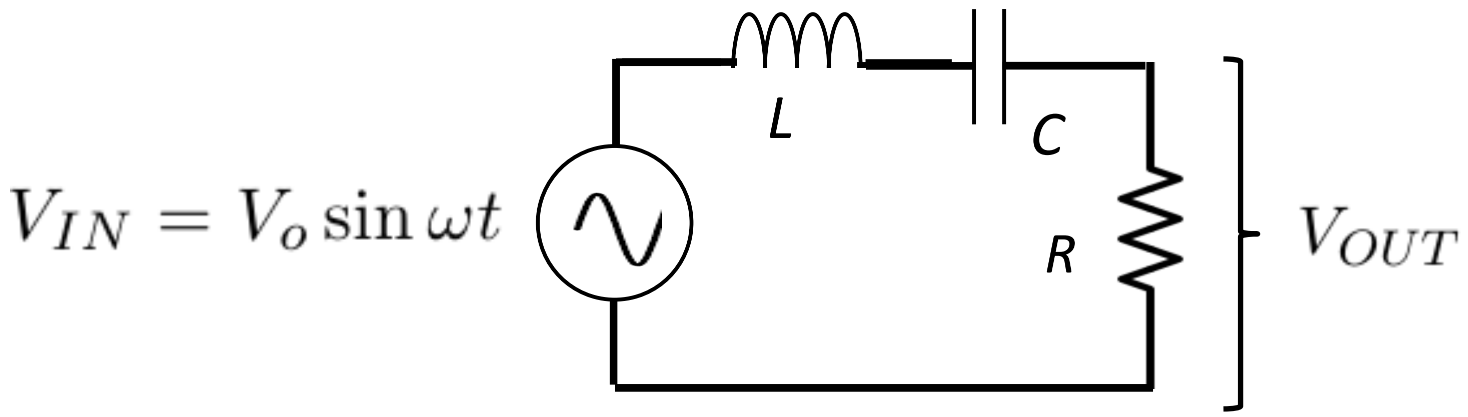
R = 100 Ω, 1 kΩ, 5 kΩ

Q = 3.1, 31, 158

$$Q = R\sqrt{\frac{C}{L}}$$



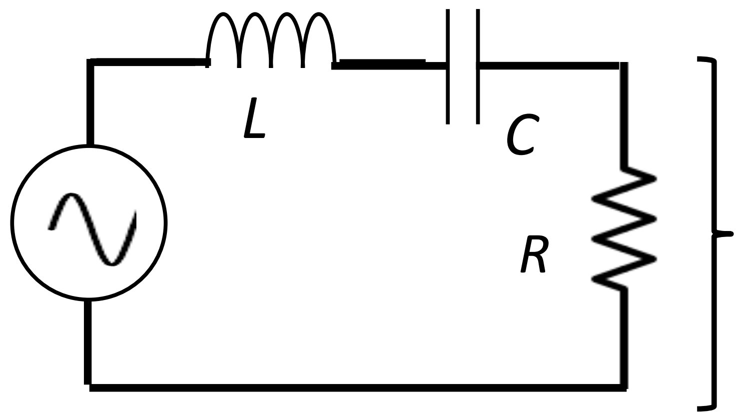
Resonance in series LC circuit



Series LC circuit

Resonance in series LC circuit

$$V_{IN} = V_o \sin \omega t$$

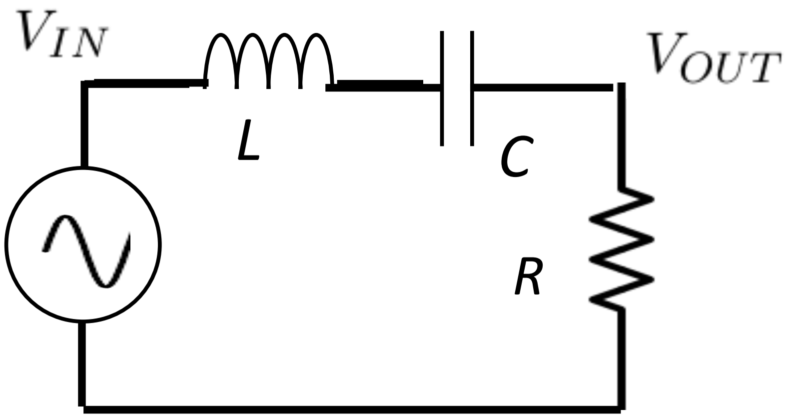


V_{OUT}

Series LC circuit

$$\frac{V_{OUT}}{V_{IN}} = \frac{R}{R + j\omega L + 1/j\omega C}$$

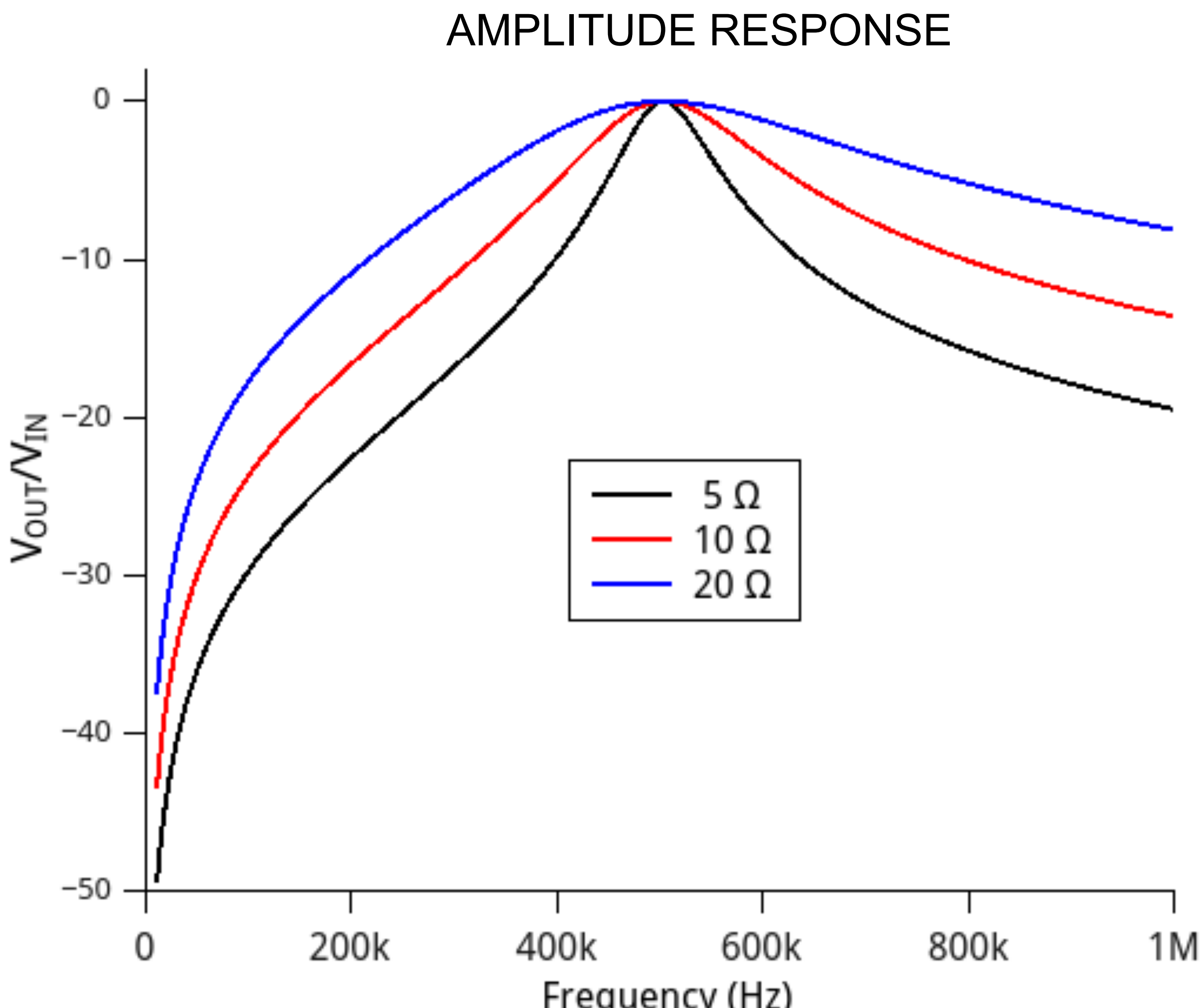
Resonance in series LC circuit



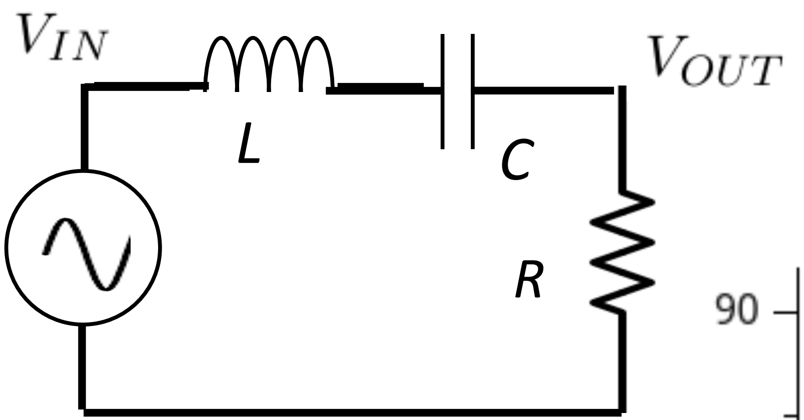
C = 10 nF
L = 10 uH
$f$res = 503 kHz

R = 5 Ω, 10 Ω, 20 Ω
Q = 6.3, 3.1, 1.6

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$



Resonance in series LC circuit



C = 10 nF
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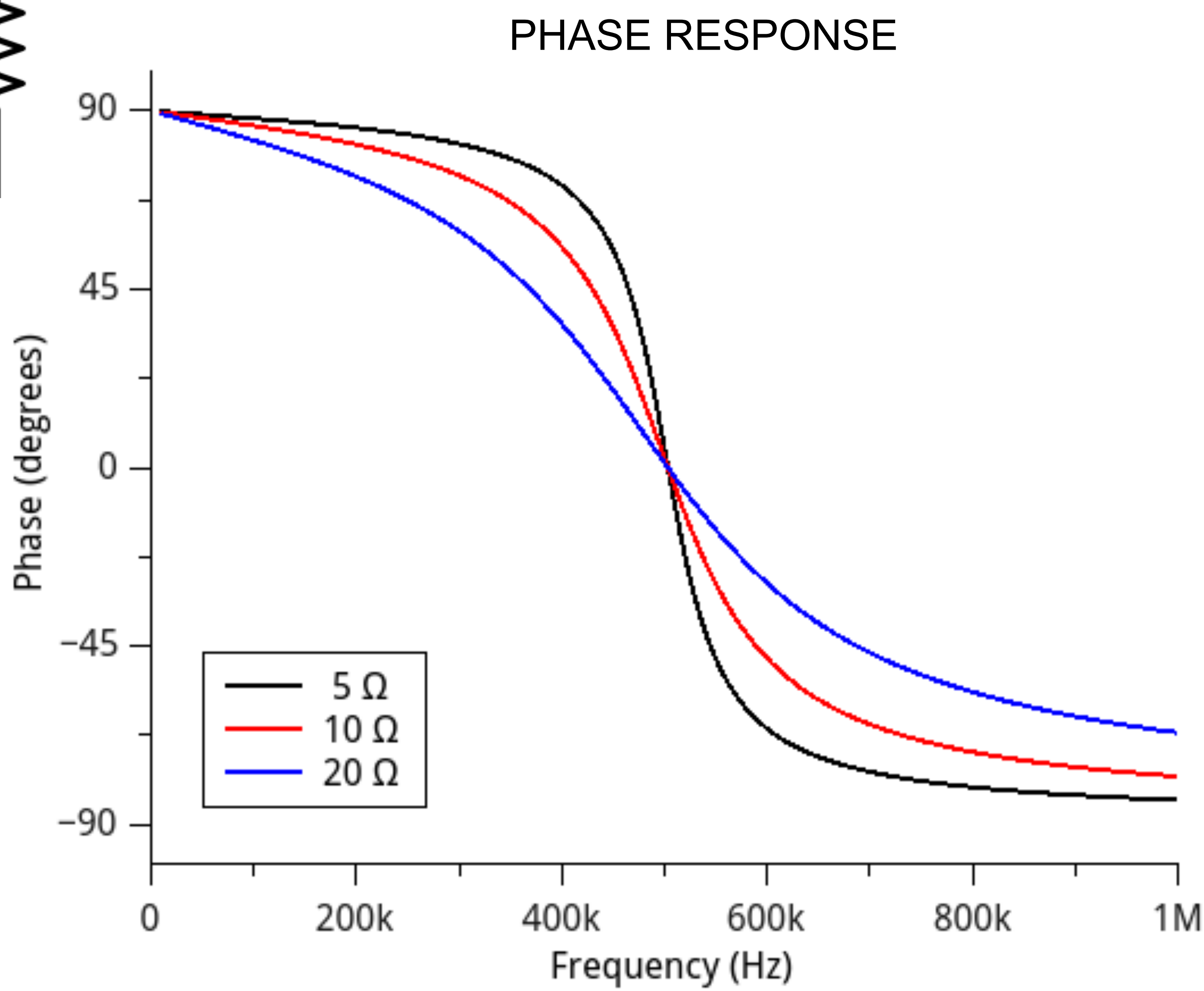
$f$res = 503 kHz

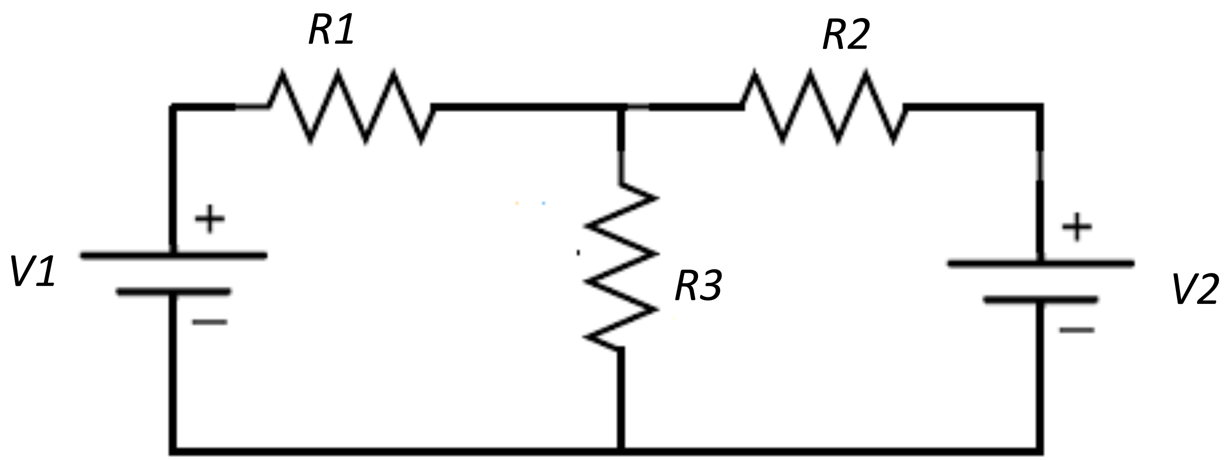


Note: Superposition Theorem

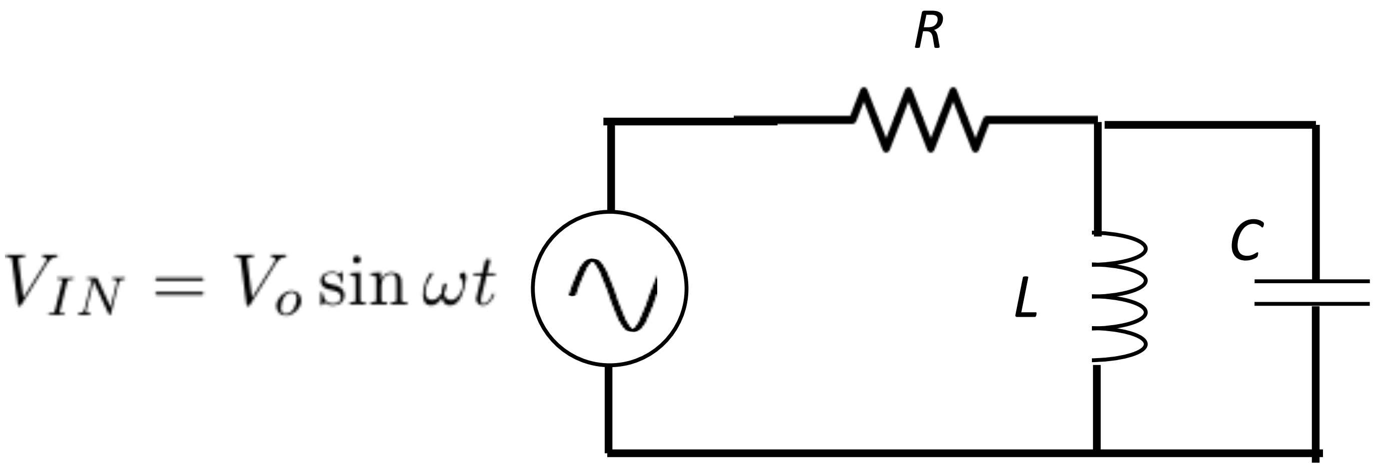
A circuit with multiple power sources can be analyzed by evaluating only one power source at a time.

1.Replace all voltage sources with short circuits (wires)
2.Replace all current sources with open circuits (breaks)

Then, the component voltages $V = \sum V_j$ *and currents are added algebraically to determine the circuit response with all power sources in effect.*



Inductor-capacitor in AC circuit: Resonance



Parallel LC circuit