Lab 5: Poles and Zeroes in AC circuits (v 1.3)

WARNING: Use electrical test equipment with care! Always double-check connections before applying power. Look for short circuits, which can quickly destroy expensive equipment.

Summary

Investigation of complex resonant circuits based on **zeros and poles** of their response function.

Learning Outcomes

- Understand the effect of **poles and zeros** in the frequency response of complex AC circuits.
- Build AC circuits with single and multiple zeros and poles based on resistors, capacitors.

Partial list of equipment needed:

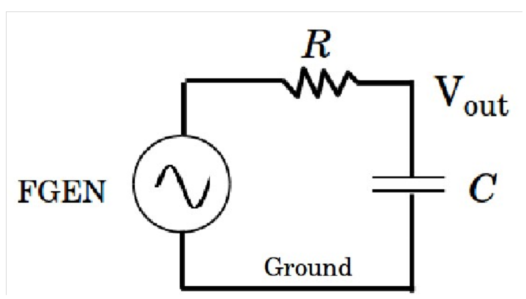
Digital oscilloscope and probes
Function generator
Breadboard and components: resistors, capacitors, etc.

Experiment/procedure

This experiment will investigate the effects of zeros and poles in the response function and how they can be designed to tailor the response of RC circuits.

RC 1-pole low pass

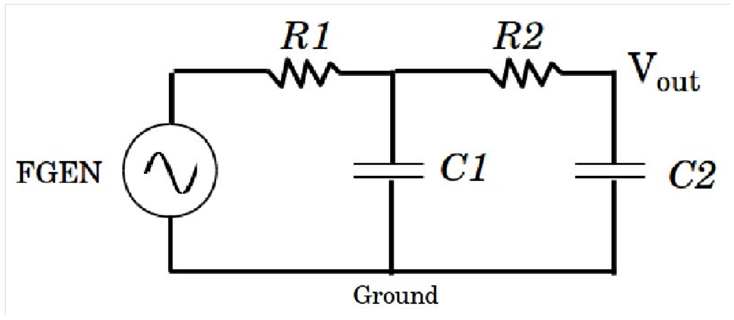
Build the following 1-pole low-pass circuit on the breadboard. Choose values of R and C to provide a cutoff frequency of your choice: $f_c = 1/(2\pi RC)$, and measure them individually.



Supply a sinusoidal input voltage V_{in} with the function generator and observe the outcome V_{out}. Scan the frequency starting well below cutoff and then increase it to verify low-pass operation. Compare it with the expected value. Investigate the decay of $G(\omega) = V_{out}/V_{in}$ after f_c in dB/decade.

RC 2-pole low pass

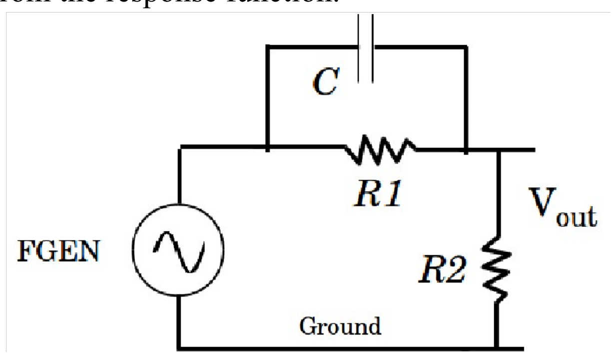
Add a second pole by modifying the circuit as shown:



where the second cutoff frequency $f_{c2} = 1/(2\pi R_2C_2)$ should be about an order of magnitude greater than the frequency f_{c1} established by R_1C_1. Collect **amplitude** and **phase** data for this two-pole filter over a suitable range of frequencies to capture the effects of the two poles. Investigate the decay of $G(\omega) = V_{out}/V_{in}$ in dB/decade over the whole range of frequencies.

RC 1-zero and 1-pole circuit

The following circuit has one zero at $f_{ZERO} = 1/(2\pi R_1 C)$ and one pole at $f_{POLE} = 1/(\frac{2\pi R_1 R_2 C}{(R_1+R_2)})$. This can (and will) be analyzed from the response function.



Set $R_1 = R_2$. This effectively will set $f_{POLE} = 2f_{ZERO}$. Collect **amplitude** and **phase** data over a suitable range of frequencies. Data should be sufficient to build the Bode plot over a suitable range of frequencies to capture the effects of the zero and pole.

Analysis

Analyze the amplitude and phase data for the last two circuits and obtain their Bode plot. Compare them with the expected frequency responses for the amplitude and phase based on theoretical models considering your measurements of the individual components for each circuit.

1-pole (low pass) circuit

Note that the response function of the RC low pass filter can be derived from the voltage divider with complex impedances for the Resistor and Capacitor, $Z_R = R$, and $Z_C = 1/jwC$, respectively. This was previously done.

2-pole circuit

Approximation: If the frequencies of the two poles are sufficiently separated, we can ignore loading effects in the two consecutive low-pass circuits. That means that the response function of the circuit $G_{2-pole}(\omega)$ can be approximated as the product of the response functions of two independent RC 1-pole circuits $G^{(1)}{}_{1-pole}(\omega) * G^{(2)}{}_{1-pole}(\omega)$.

1-zero and 1-pole circuit

Derive the frequency response function $G(\omega)$ for the "*1-zero and 1-pole circuit*". This can be easily obtained from a "voltage divider" with complex impedances.

In your lab notebook, include the derivations of the equations for the analysis, plots of results, and discussion of results. The "2-pole circuit" and "1-zero 1-pole circuit" should be analyzed for amplitude and phase and compared to experimental data. You will need to use a program such as MatLab, C++, Python, etc. for the analysis of the resonant circuit. This program is the same, or very similar, to the ones used in previous lab.

Note: Recall the definitions for amplitude (dB) and phase (radians) in the Bode data:

$$\text{Amplitude (dB)} = 20\log_{10}|G|$$

$$\text{Phase (rad)} = \text{atan}\left[\frac{\text{Im}(G)}{\text{Re}(G)}\right]$$

where $G = V_{OUT}/V_{IN}$.