

Lab 4: Frequency response and resonant circuits (version 1.3)

WARNING: Use electrical test equipment with care! Always double-check connections before applying power. Look for short circuits, which can quickly destroy expensive equipment.

Summary

Introduction to frequency response of RLC circuits, inductor, and resonance circuits.

Learning Outcomes

- Understand the relationship between charge and voltage on the capacitor and inductor.
- Build AC circuits with resistors, capacitors, and inductors.
- Realize and analyze measurements of location and width of resonances.
- Understand and Analyze Bode plots and the dB scale.

Partial list of equipment needed:

Digital oscilloscope and probes

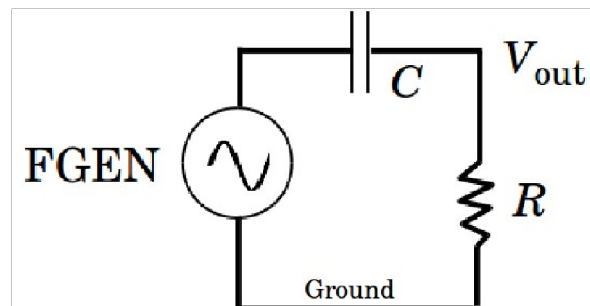
Function generator

Breadboard and components: resistors, capacitors, and inductors.

Experiment/Procedure

High pass filter

Build the following circuit on the breadboard. Choose values of R and C to provide a cutoff frequency $f_c = 1/(2\pi RC)$ in the range 5-10 kHz. Measure components and record the values before you assembly the circuit. Use the function generator to generate a Sine waveform with amplitude = 500 mVpp, and a DC Offset = 0, with a frequency well above the cutoff. Reduce the frequency below cutoff and verify high pass operation.



Measure the amplitude of V_{out} **relative** to the V_{in} as a function of frequency using the two channels of the oscilloscope. Use a frequency range starting well below f_c and ending well above it to provide enough data to map out the response. Collect data at a minimum 12 individual frequencies for amplitude. More data is better, but not required. Plot the amplitude response, where the frequency axis is on a log scale and the amplitude in dBs = $20 \log_{10}(|V_{OUT}/V_{IN}|)$.

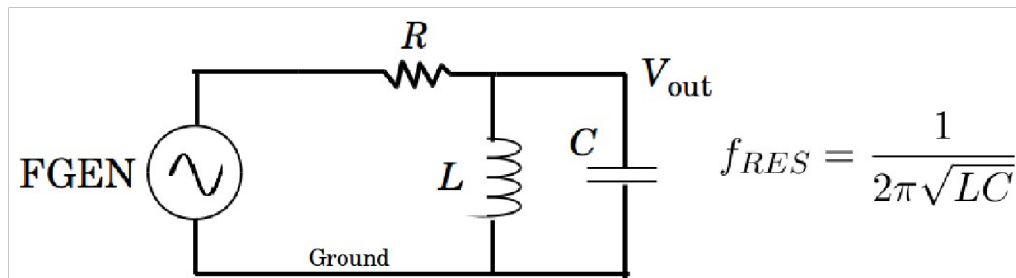
Resonant Circuits

RLC circuits can exhibit sharp resonances in their response function $G(\omega) = V_{out}/V_{in}$. The frequency and sharpness of these resonances depend on the specific values of R , L , and C . Below you will study two RLC resonant circuits: Series and Parallel circuits, and investigate their response $G(\omega)$ as a function of frequency.

You will collect (amplitude and phase) data as a function of frequency to build Bode plots for each circuit, where the frequency axis is on a log scale and the **amplitude** in dBs = $20 \log_{10}(|V_{OUT}/V_{IN}|)$. The relative **phase between V_{in} and V_{out}** can be measured by measuring the relative time delay Δt with the markers in the oscilloscope and compare it with the period $T = 1/f$ of the sinusoidal wave at that frequency, which corresponds to a full wave phase cycle of 360 degrees.

Parallel RLC circuit

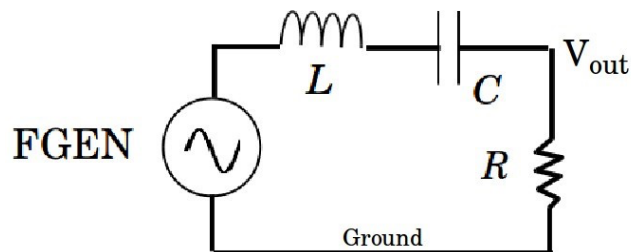
Build the following RLC circuit. Select values of L and C to produce a resonant frequency f_{RES} in the range 5-20 kHz (not critical, can be higher). The resistor value R will establish the width of the resonance peak that can be seen in the frequency domain. Experiment with this value. R in the range 100-200 Ω is a good starting point, with $C \sim 0.1 \mu\text{F} - 5 \mu\text{F}$. If R is too small, the resonance peak will be very broad and wash out; if it is too big, in principle the peak will be very sharp and approach a delta function. Be sure to independently measure R , L , and C , which will be needed for the analysis of this lab.



For data collection, choose a span of frequencies so that the resonance peak is well resolved. Make sure there are enough points so that you can identify the -3 dB points (0.707 of the peak magnitude) to obtain the circuit Q . Perform measurements of amplitude and phase to build the Bode diagram.

Series RLC circuit (Band-Pass Filter)

Build the following circuit, which is another resonant circuit, but choose L and C to get a much higher resonant frequency, i.e. $f_{RES} \approx 50$ -300 kHz. Reduce the amplitude of the input signal to 100 mV and use a value of R in the range 5-10 Ω with C in the 5-50 nF range.



Choose a frequency range and number of data points to resolve the resonance peak with enough resolution. Perform measurements of amplitude and phase to build the Bode diagram.

Analysis and Writeup

For the analysis in your lab notebook, derive/use the response functions $G(\omega) = V_{out}/V_{in}$ for the RC (high pass) and the two resonant RLC circuits to model their amplitude and phase responses. The response functions can be easily obtained from the general expression of a voltage divider with complex impedances for each element:

$$\begin{aligned}Z_R &= R \\Z_L &= j\omega L \\Z_C &= 1/j\omega C.\end{aligned}$$

From these, you can obtain the expressions for the frequency-dependent amplitude and phase response for the circuits.

Make six theoretical plots (2 for each circuit) showing amplitude (dB) and phase (degrees) as a function of frequency (use a log-10 scale for the x-axis). Plot them together with your data to compare them with your observations. Include your equation formulas on each plot and the component values. (note that the phase response for the RC high pass is not required).

Determine the theoretical value of Q for both resonant circuits, where $Q=f_0/\Delta f$, and where Δf is the frequency range between the -3 dB locations, and compare with your experimental data.

Use MATLAB or Python to plot your data on the same plot (amplitude and phase). Paste this into your lab book. Discuss the results and comparison with theory.