Lab 3: AC Low-pass filters (version 1.3)

WARNING: Use electrical test equipment with care! Always double-check connections before applying power. Look for short circuits, which can quickly destroy expensive equipment.

Summary
Introduction to the capacitor, reactive circuits, and the low-pass filter.

Learning Outcomes

- Understand the relationship between charge and voltage on the capacitor and inductor.
- Build an AC circuit with resistors, capacitors & inductors.
- Realize and analyze measurements of voltage drop and phase shifts.

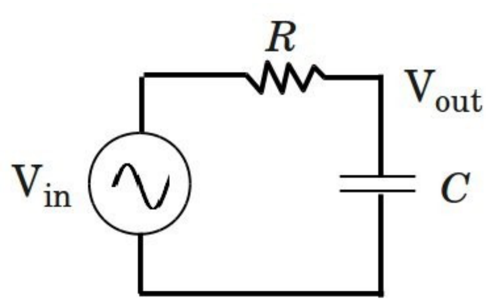
Partial list of equipment:
Digital oscilloscope
Function generator
Breadboard and components: Resistors: 200 Ω and 1 kΩ; Capacitor: 100 nF; and Inductor: 3.9 mH. (*Exact values are not critical, substitute as needed*.)

Frequency response and complex impedance
In this lab, we will study the time-dependent response of *low-pass* R-C and L-R circuits to time-varying waveforms. Specifically, we will investigate the frequency dependent behavior of a circuit and phase shift on AC signals. The behavior can be modeled using complex impedance of different components. For example, assuming a sinusoidal wave $V(t) = V_0 sin\,(\omega t)$,

- $Z_R = R$,
- $Z_C = 1/j\omega C$,
- $Z_L = j\omega L$, where $j = \sqrt{-1}$.

Experiment/procedure

1. Low pass RC filter
Build the following RC circuit (nominally R = 200 Ω and C = 0.1 μF). Specific values are not critical, but you need to measure each component before you assemble circuit. Setup the function generator to produce a 1 Vp-p sine wave at 1 kHz. Using the oscilloscope, observe V_{in} on CH1 and V_{out} on CH2.



__Note 1:__ The oscilloscope amplitude may differ from the function generator setting by a factor of 2. This is because the output impedance of the function generator may be set to 50 Ω. Change it to high impedance (High Z), if necessary.

Measurement of the signal waveforms

Measure the **amplitude** and **phase** of V_{out} **relative** to V_{in} as a function of frequency for at least ~10 different frequencies using the oscilloscope. A frequency range *f* between 1kHz and 100-200 kHz should provide enough data to map out the response, depending on the choice of R and C. However, the frequency span range chosen should show the drop off in output-to-input signal ratio (V_{out}/V_{in}) with increasing frequency to less than $V_{out}/V_{in}<0.1$.

For relative phase measurements, the time delay between two waveforms can be directly converted to a phase difference. This is because the period *T* of the waveforms is known $T=1/f$.

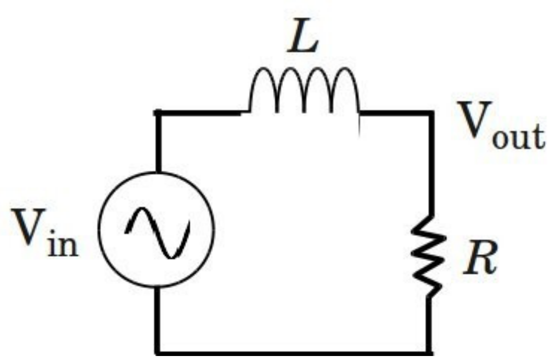
Calculate the characteristic cut off frequency of the R-C circuit $f_c = w_c/2\pi = 1/2\pi RC$. A low-pass filter reduces the output V_{out} as the frequency increases, with a reduction of power by a factor of two at f_c. In your lab book, you will plot and analyze your data, and compare to theoretical expectations.

B. Response to square waves (charge/discharge of a capacitor)

Investigate the response of the RC circuit to an input V_{in} square wave and observe the output of the circuit. V_{out} has a time response showing the process of charging and discharging of the capacitor. Determine the time constant for this charging/discharging process. Compare your measurements with theoretical predictions based on your measured resistance and capacitance.

2. Low-pass filter with inductor

Build the low-pass filter implemented with an inductor and resistor arranged as follows:



Use R~1 kΩ and an inductor in the range 3-5 mH to produce a cutoff frequency near 40 kHz (not critical) according to $f_c = w_c/2\pi = R/2\pi L$. Perform the same set of measurements as with the R-C filter, but adjust your data collection frequency range to account for the higher cutoff frequency. The goal is to show the frequency response roll off with data points above and below the cutoff frequency. Obtain data to analyze both amplitude and phase.

Analysis:

For the analysis in your lab notebook, use the response functions $G(\omega) = V_{out}/V_{in}$ for the R-C and L-R circuits to model the amplitude and phase responses of the circuits you built. Using the fact that the response function $G(\omega)$ is a complex number $a + jb$, from $G(\omega)$ you can obtain the frequency-dependent amplitude and phase response for the R-C and L-R circuits.

1. Plot the amplitude and phase response of $G(\omega) = V_{out}/V_{in}$ for the R-C circuit for your data. Express the amplitude response as decibels $20log_{10}(|ratio|)$. Display the phase data in degrees (not radians). The x-axis of both plots should be frequency *(f)* in units of Hz or kHz.

2. Repeat the analysis described in (**1**) for the R-L circuit.

3. The four data sets generated in (**1**) and (**2**) have corresponding theoretical curves from the response functions $G(\omega) = V_{out}/V_{in}$ using your measured values for R, L, and C. Add the theory curves to the four plots from **1**) and (**2**) and compare with your observations. Include your equations on each plot and the component values.

Use MATLAB or Python to plot your data on the same plot. Paste this into your lab book and discuss the results.

Note 3: *The R-C circuit is one of the most important concepts in electronic instrumentation, as any circuit always has stray capacitance and input impedance, including the oscilloscope. If you look at the input connection of any oscilloscope, the input impedance associated with it will be labeled. A typical scope may have a 1 MΩ input resistance in parallel with 20 pF of stray (unintentional but unavoidable) capacitance. This means the scope may act as a low-pass filter with a cutoff frequency of around 8 kHz. The cutoff can be increased to nearly 160 MHz by using a 50 Ω BNC terminator, which will be in parallel with the internal 1 MΩ resistance. This value is chosen because it is comparable with the characteristic impedance of coaxial cable. When a 50 Ω terminator is used, however, the measured voltage levels will drop accordingly.*