Lab 5: AC circuits (III)

Poles and Zeroes

$$\frac{V_{OUT}(\omega)}{V_{IN}(\omega)} = G(\omega) = \frac{N(\omega)}{D(\omega)}$$

$$\text{Numerator: } N(\omega) = (\omega_{N1} + j\omega)(\omega_{N2} + j\omega) \times \ldots (\omega_{Nm} + j\omega)$$

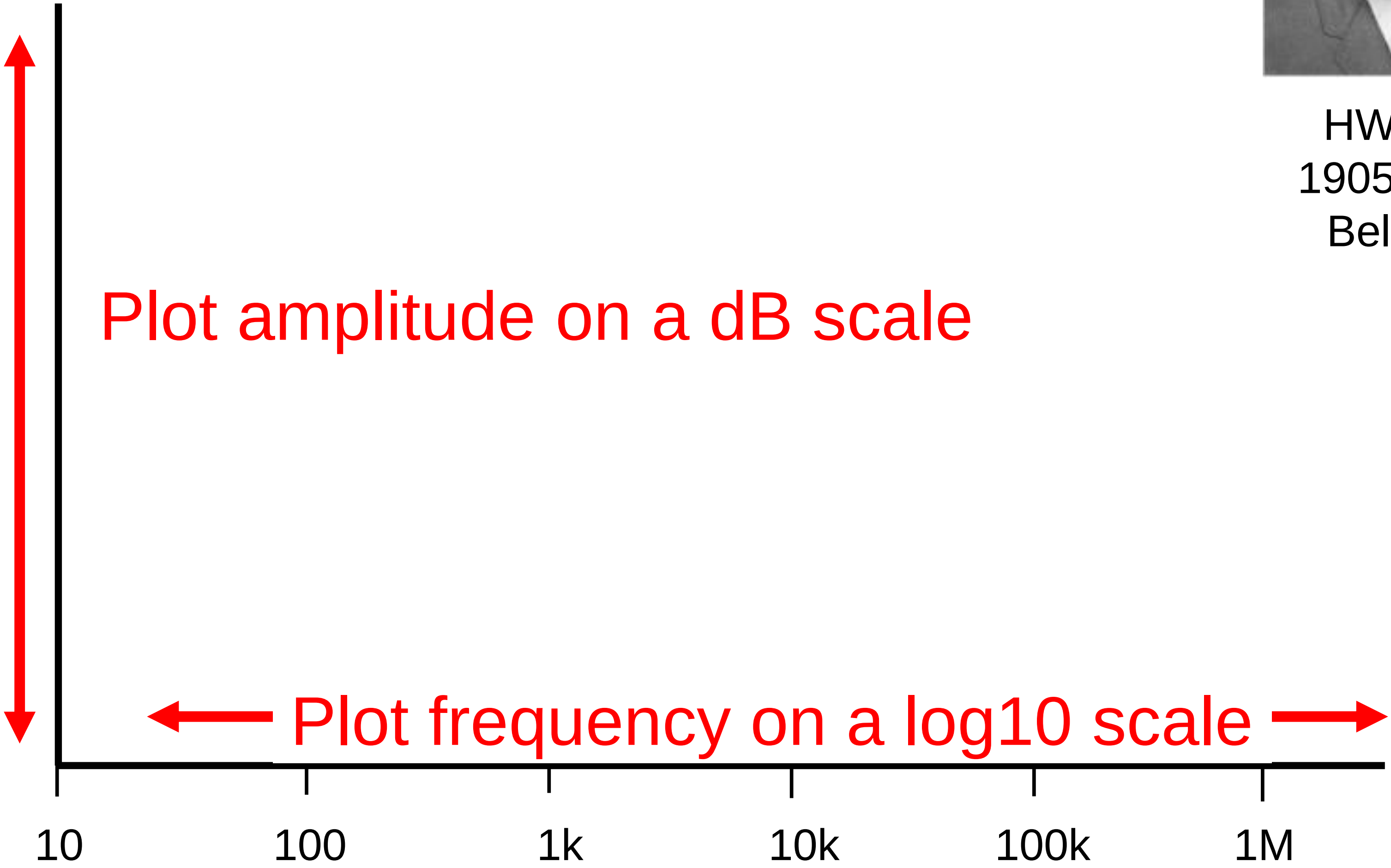
$$\text{Denominator: } D(\omega) = (\omega_{D1} + j\omega)(\omega_{D2} + j\omega) \times \ldots (\omega_{Dm'} + j\omega)$$

$$m \leq m'$$

Zeroes: Frequencies (ω_N) that make the numerator zero
Poles: Frequencies (ω_D) that make the denominator zero

The Bode Plot for |*G*(ω)|

HW Bode
1905—1982
Bell Labs



POLES **and** ***ZEROES*** **change the slopes on the Bode Plot**

Amplitude response

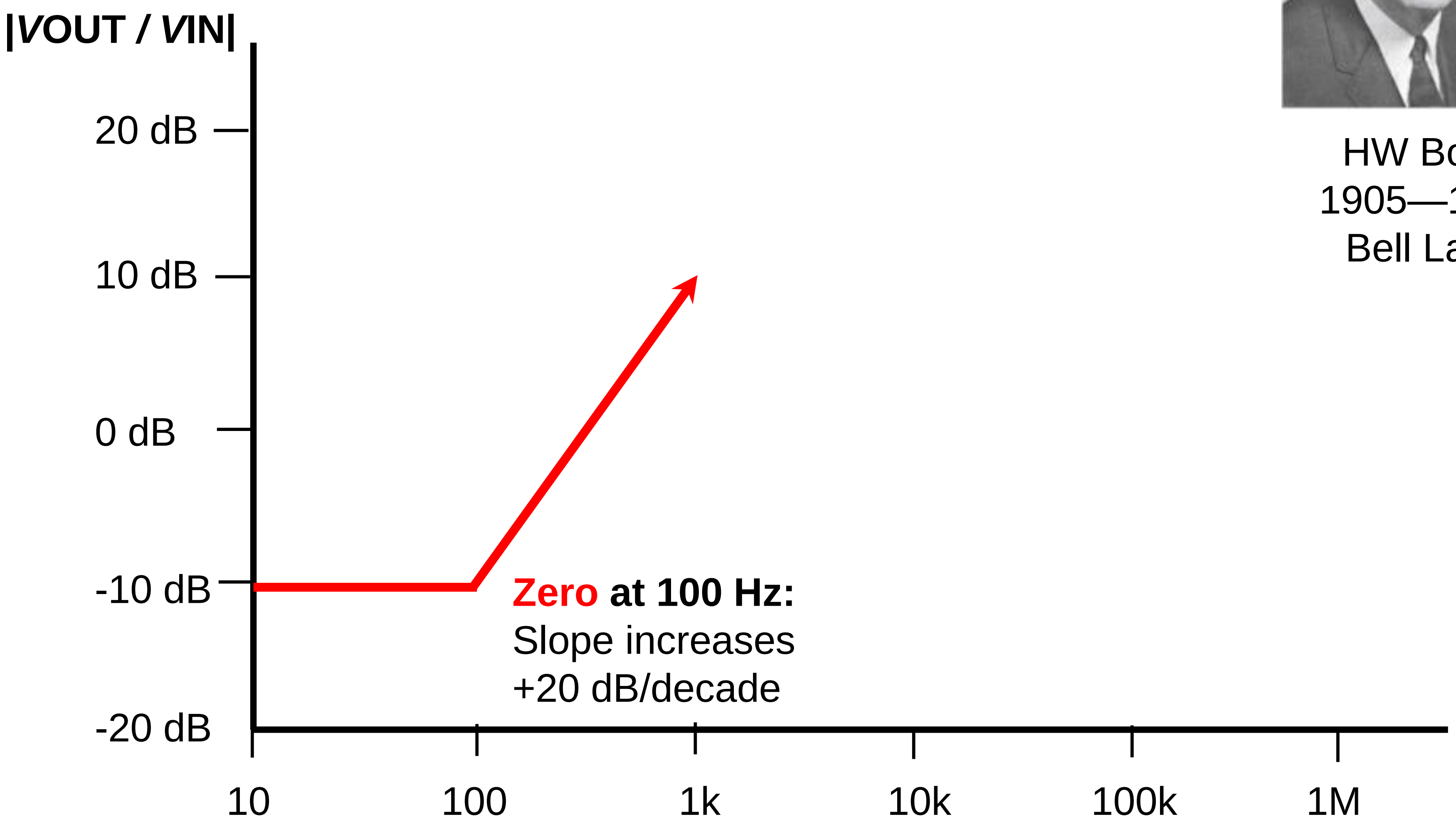
ZERO: + 20 dB/decade
POLE: – 20 dB/decade

Phase response

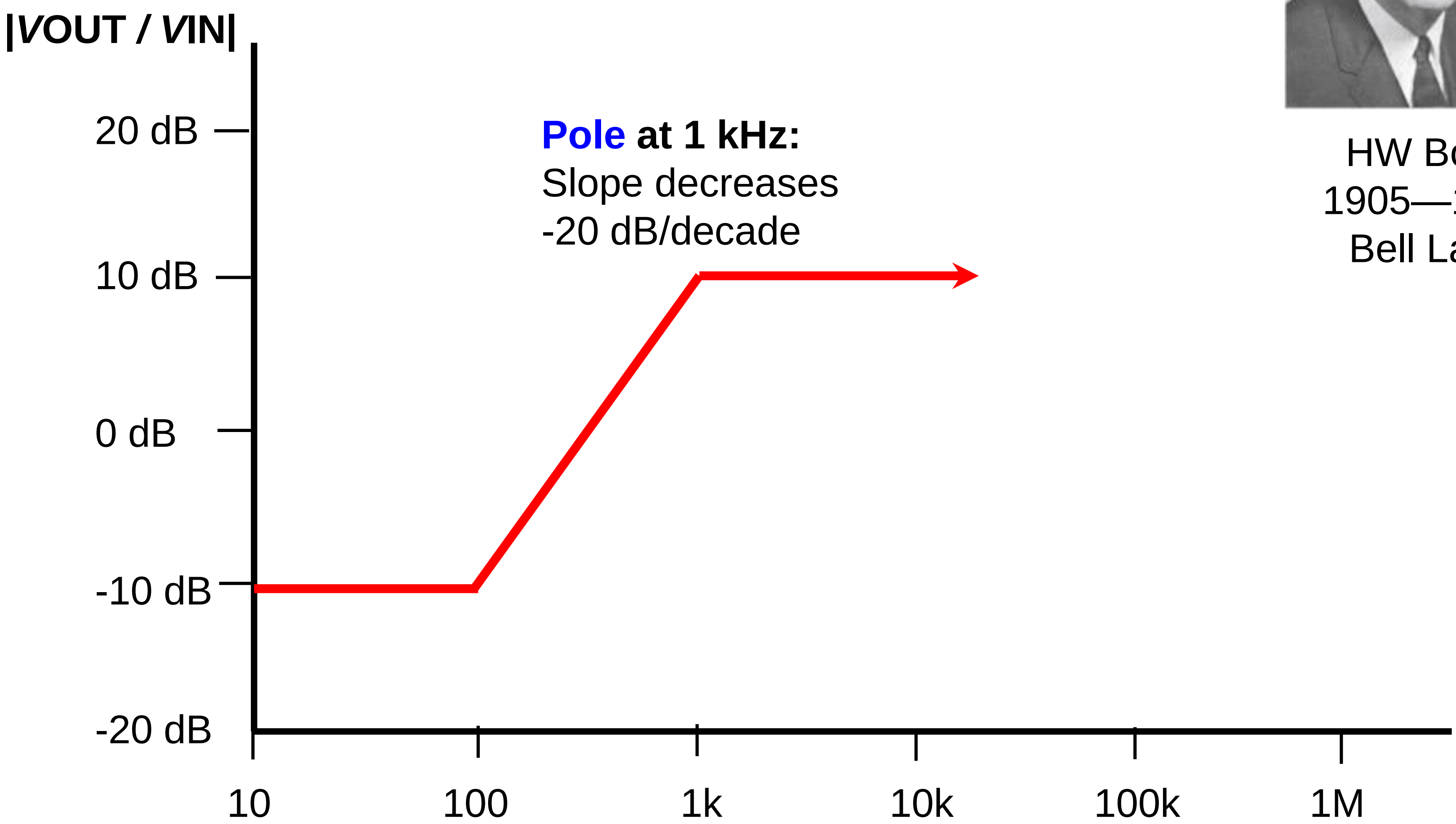
ZERO: + 45 deg/decade
POLE: – 45 deg/decade

POLES and ZEROES dramatically affect the shape of a Bode plot

HW Bode
1905—1982
Bell Labs

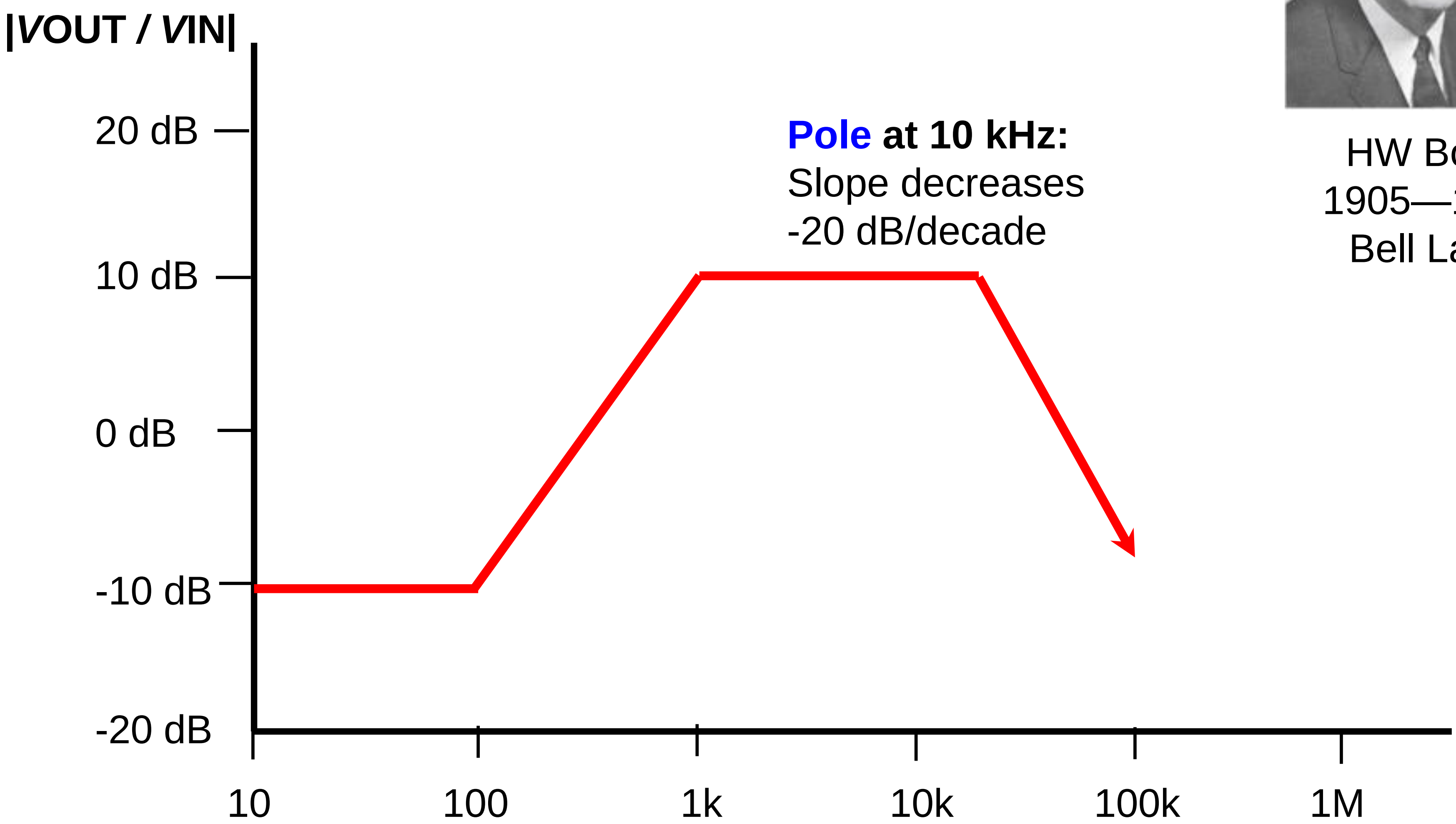


POLES and ZEROES dramatically affect the shape of a Bode plot



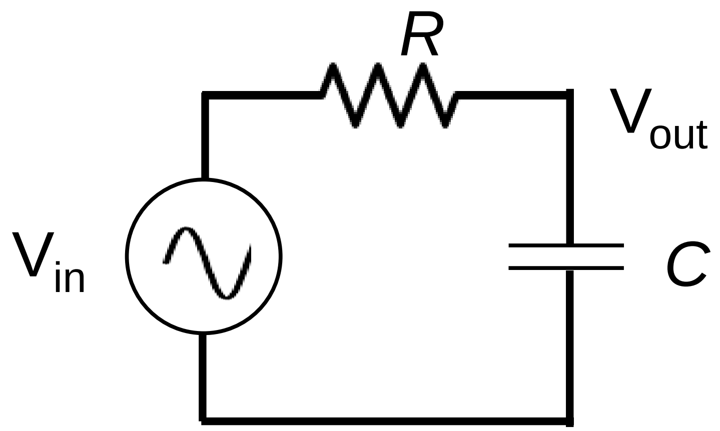
HW Bode
1905—1982
Bell Labs

POLES and **ZEROES** dramatically affect the shape of a Bode plot



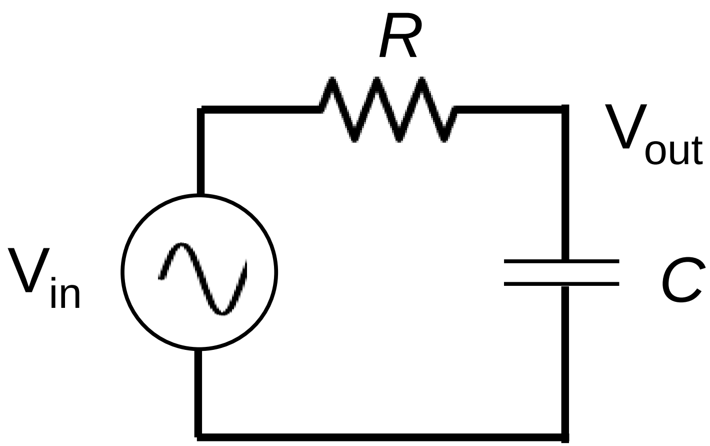
HW Bode
1905—1982
Bell Labs

Example 1: Low-pass filter



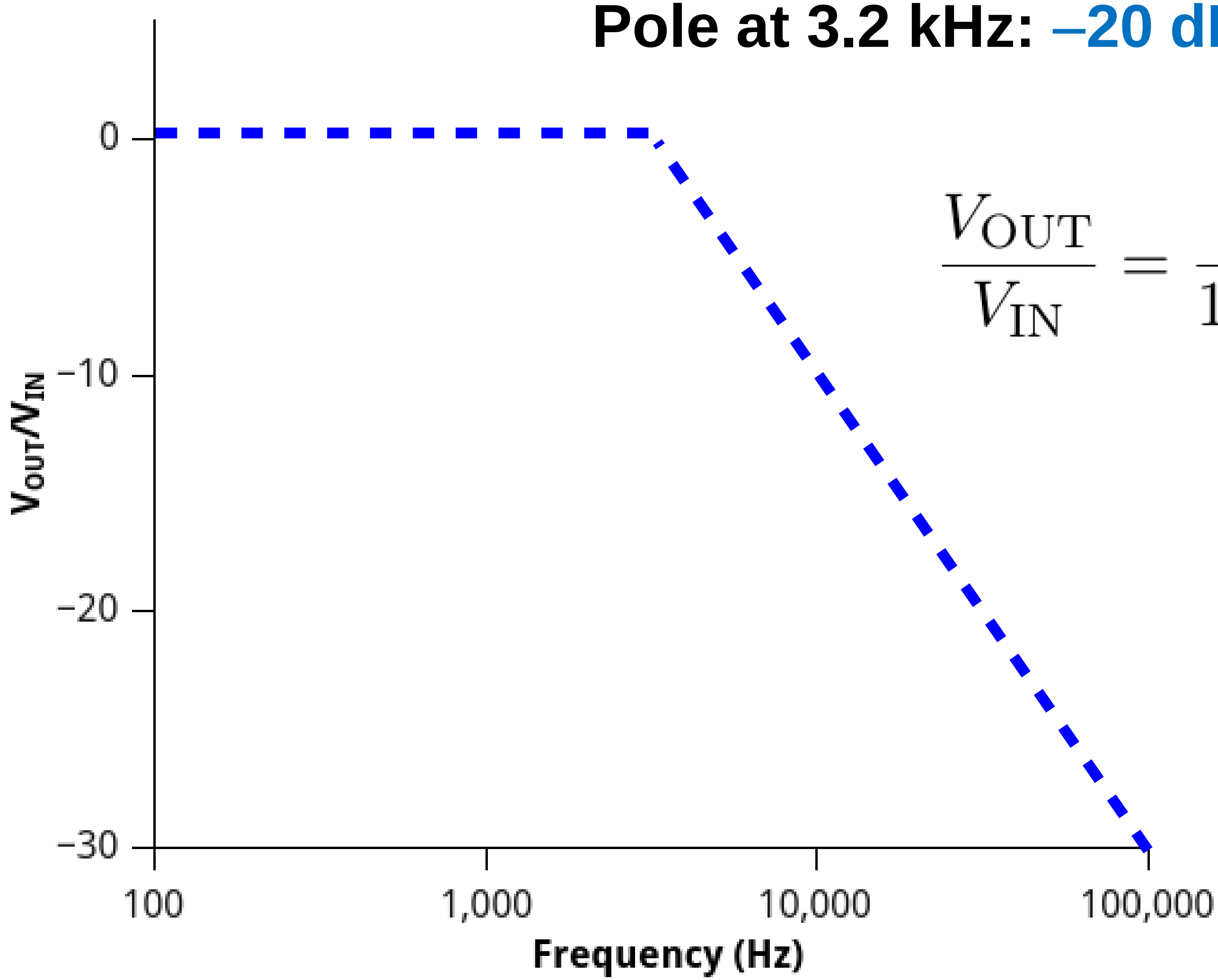
$$\frac{V_{out}}{V_{in}} = \frac{Z_C}{Z_R + Z_C} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC}$$

Pole in denominator

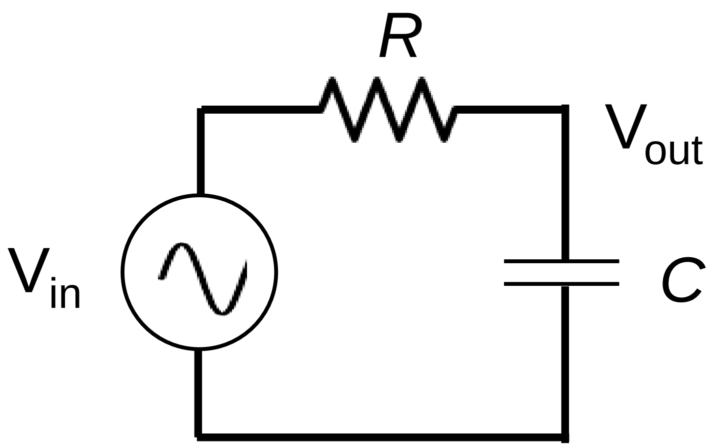


R = 5000 Ω
C = 10 nF
f = 3.2 kHz

Pole at 3.2 kHz: –20 dB/decade



$$\frac{V_{\mathrm{OUT}}}{V_{\mathrm{IN}}} = \frac{1}{1 + j\omega RC}$$



R = 5000 Ω
C = 10 nF
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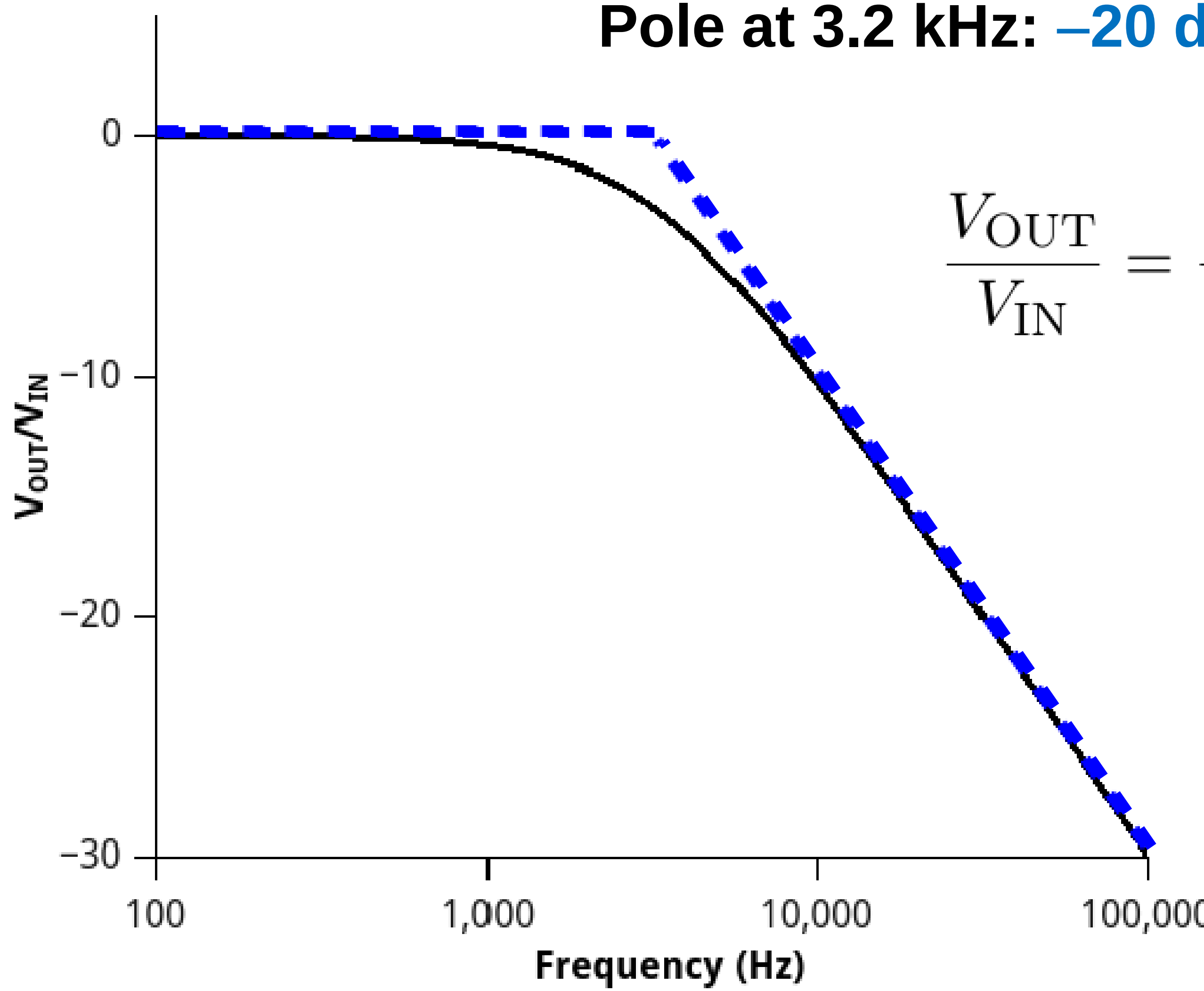
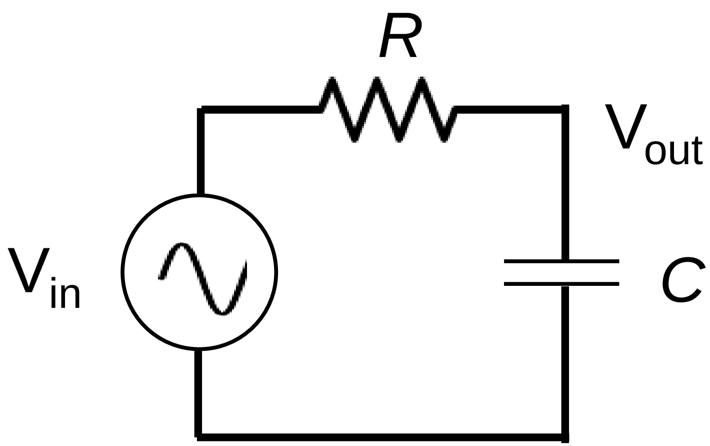
Pole at 3.2 kHz: –20 dB/decade



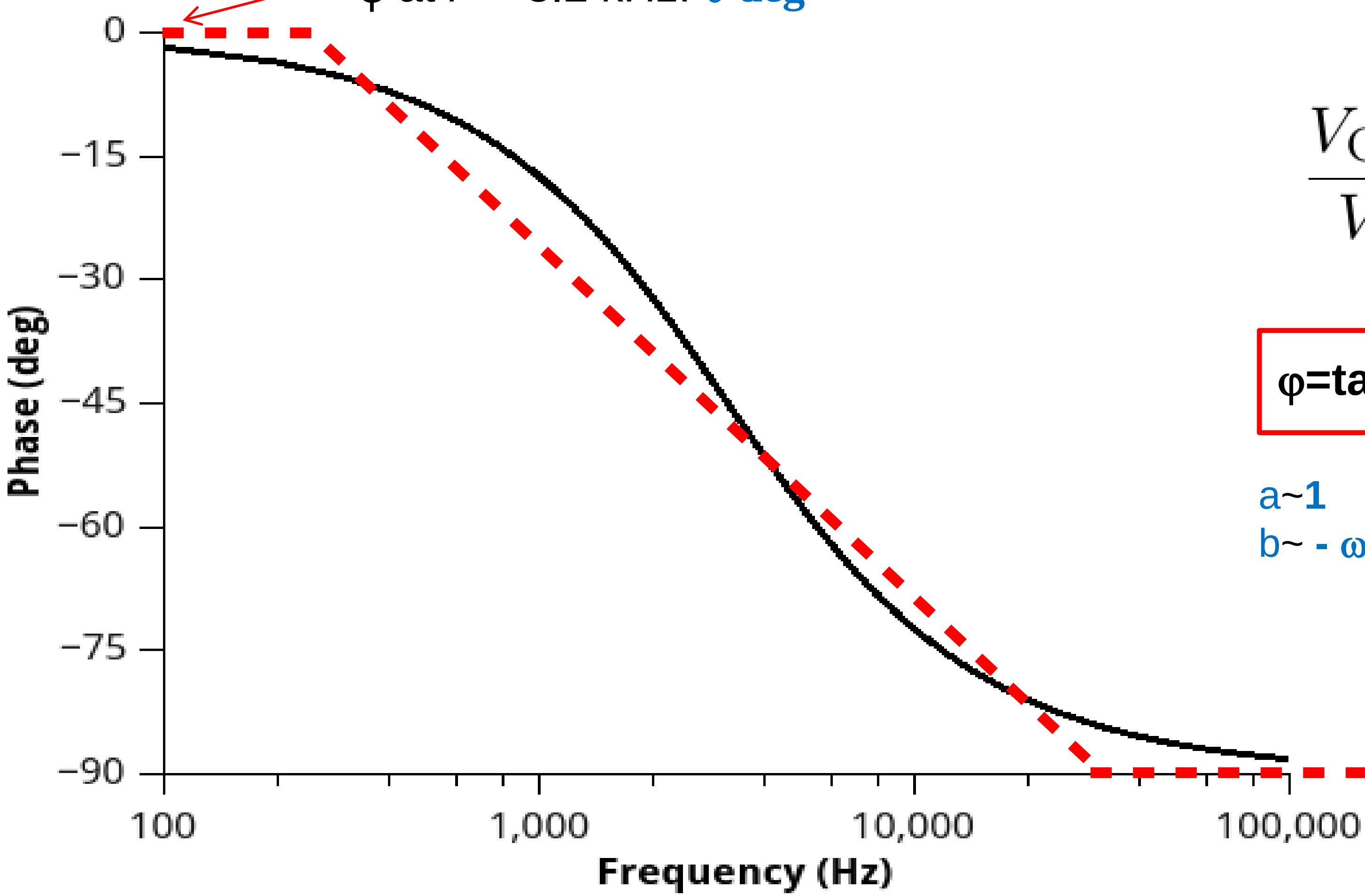
$$\frac{V_{\mathrm{OUT}}}{V_{\mathrm{IN}}} = \frac{1}{1 + j\omega RC}$$



$R = 5000\ \Omega$
$C = 10\ \text{nF}$
$f = 3.2\ \text{kHz}$

Pole at 3.2 kHz: –45 deg/decade

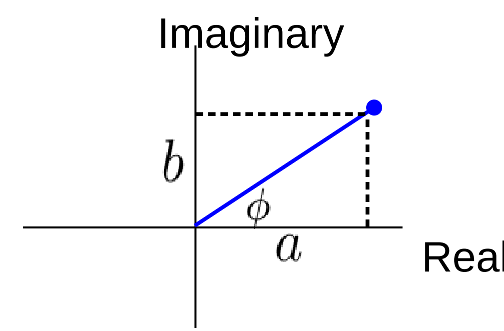
φ at *f* <<3.2 kHz: **0 deg**



$$\frac{V_{\mathrm{OUT}}}{V_{\mathrm{IN}}} = \frac{1}{1 + j\omega RC}$$

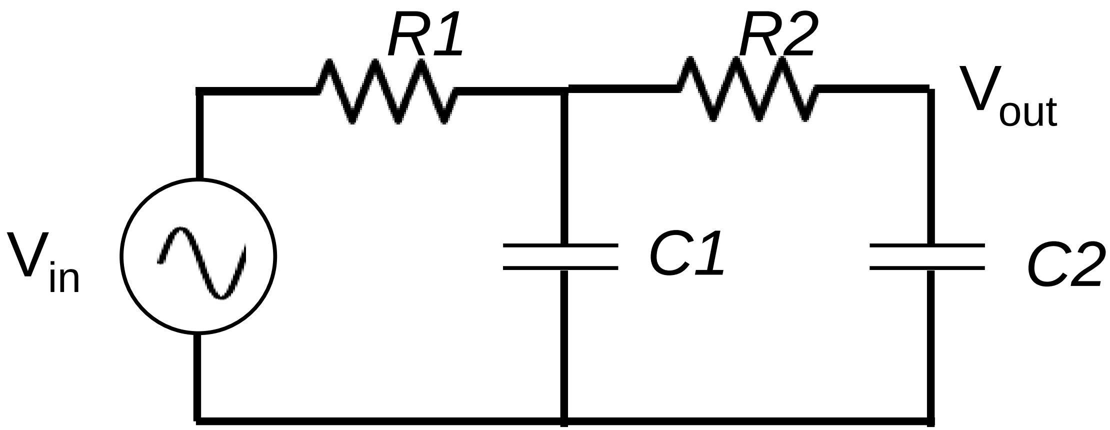
$\varphi = \tan^{-1}(b/a) = \tan^{-1}(-\omega RC/1)$

a~**1**
b~ **- ωRC**



φ at *f* >>3.2 kHz: **–90 deg**
(f=10 f_{res})

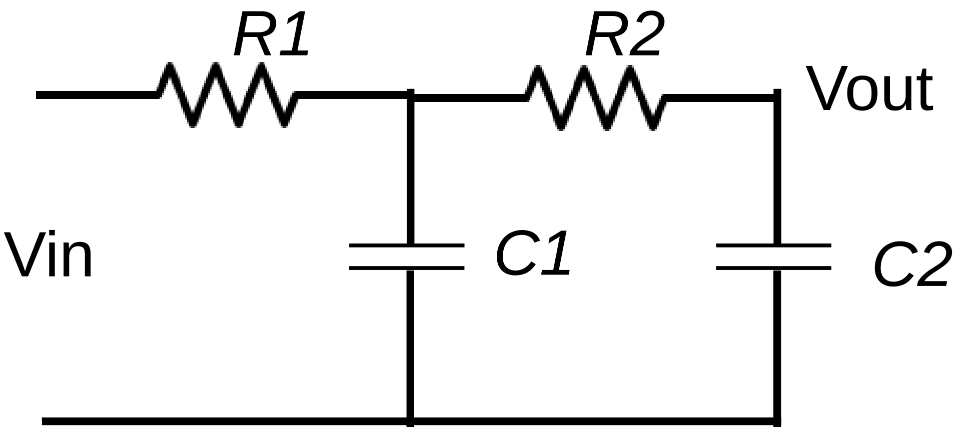
Example 2: 2-pole low-pass filter



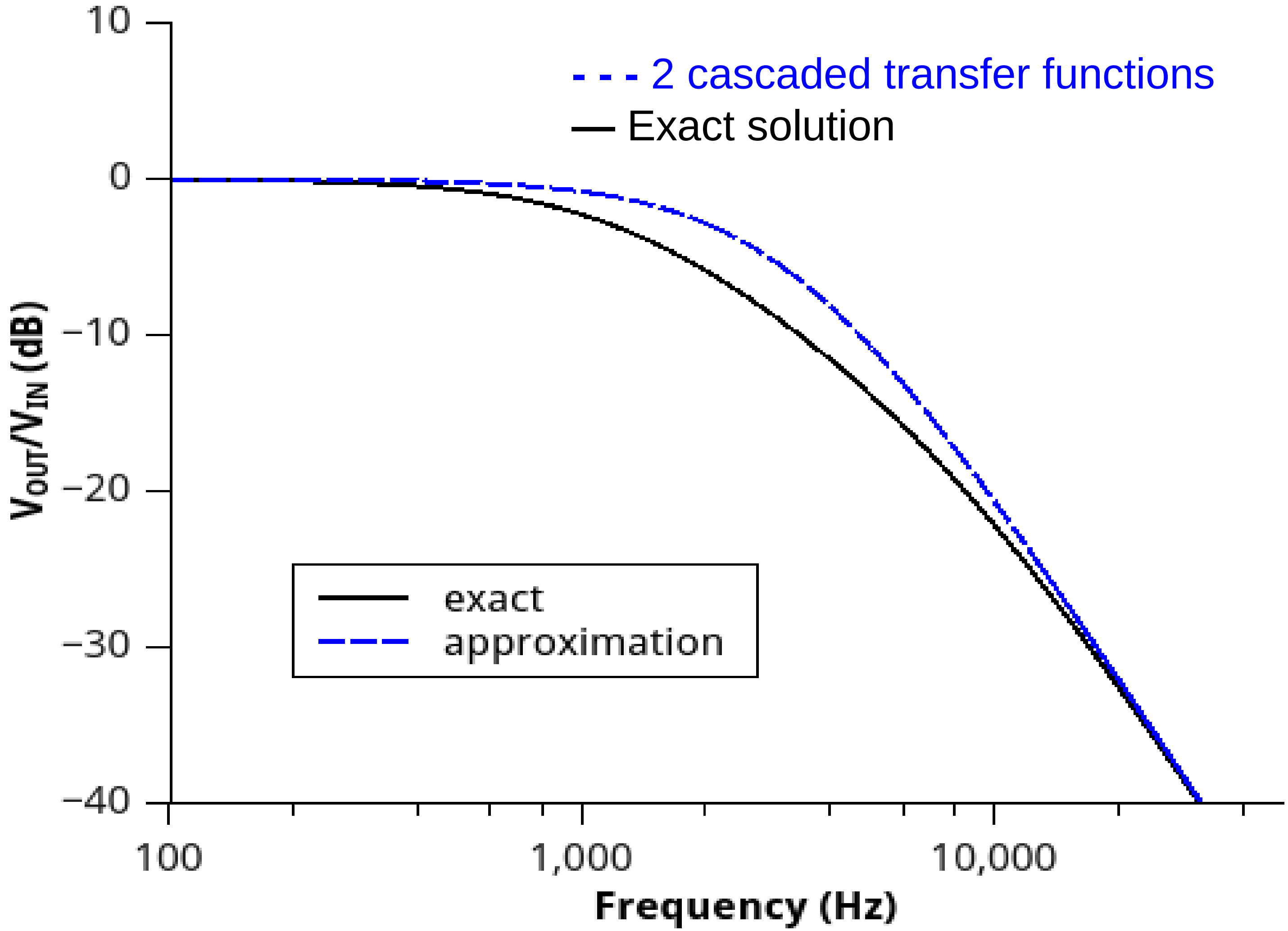
Assume "no loading"

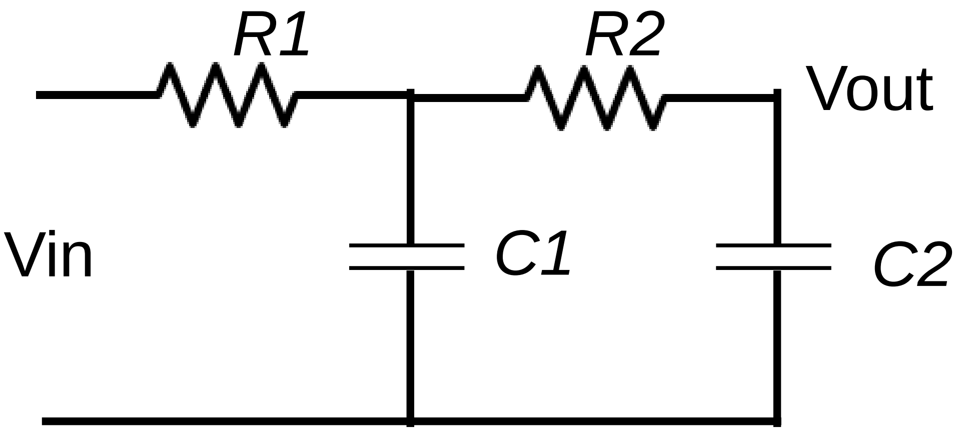
$$\frac{V_{\mathrm{OUT}}}{V_{\mathrm{IN}}} \approx \left[\frac{1}{1+j\omega R_1 C_1}\right]\left[\frac{1}{1+j\omega R_2 C_2}\right]$$

1st pole **2nd pole**



R1 = 5000 Ω *R2* = 500 Ω
C1 = 10 nF *C2* = 3 nF
f = 3.2 kHz *f* = 10.6 kHz

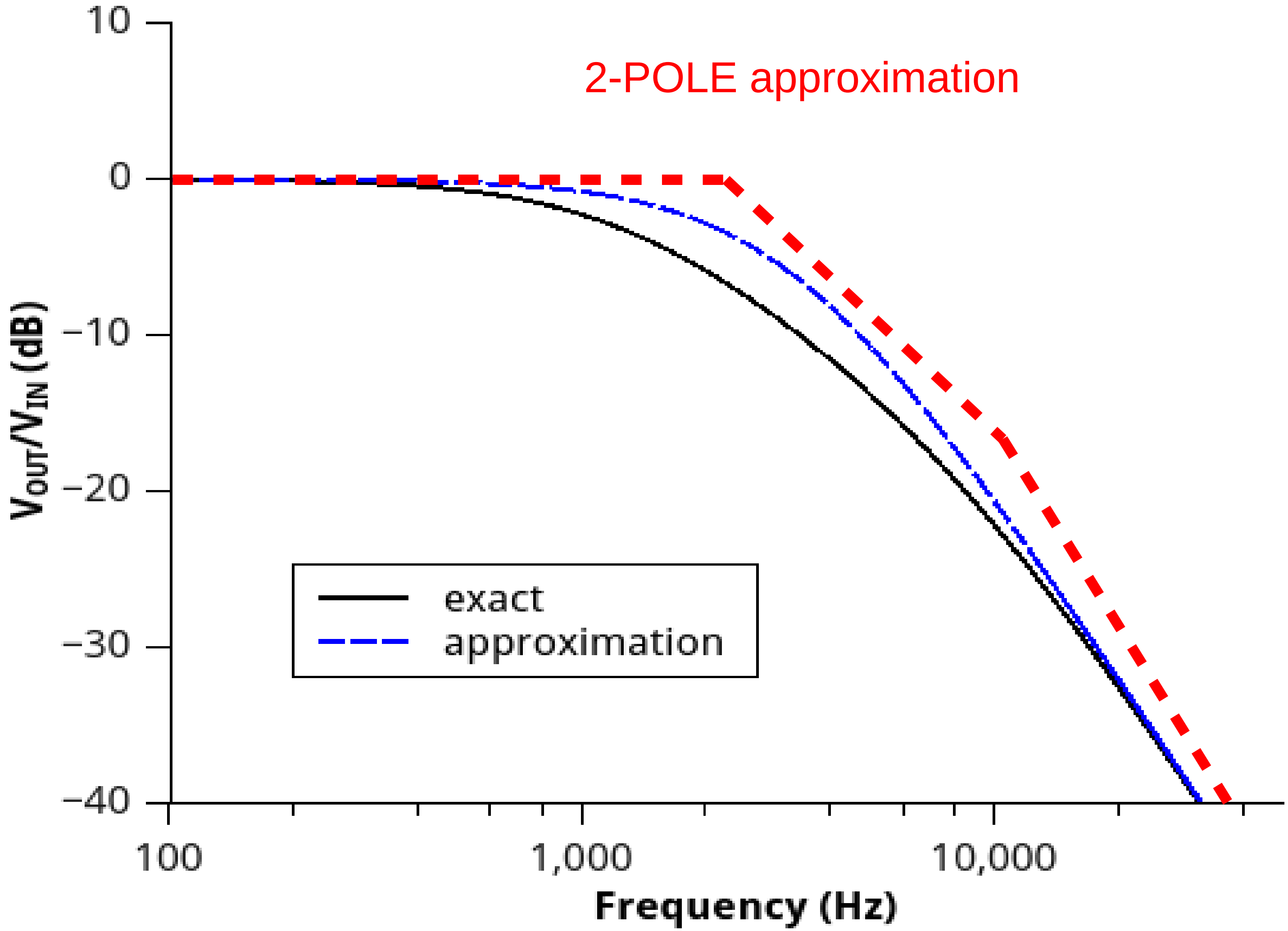


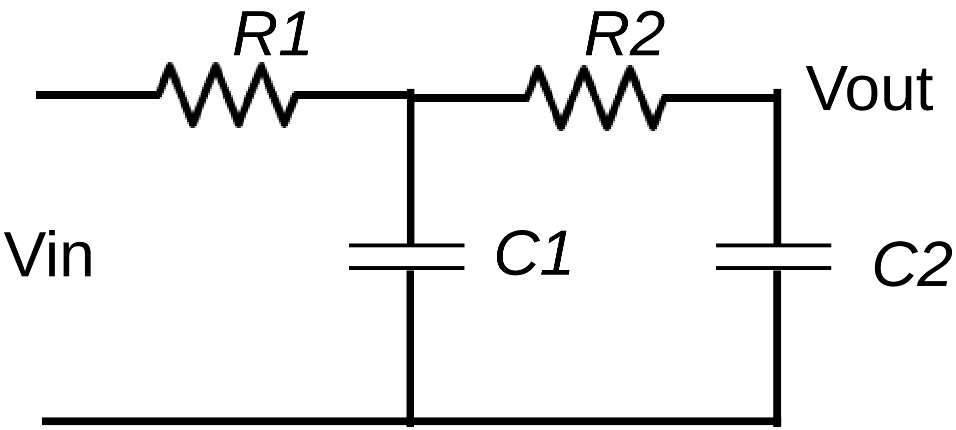


$R1$ = 5000 Ω $R2$ = 500 Ω

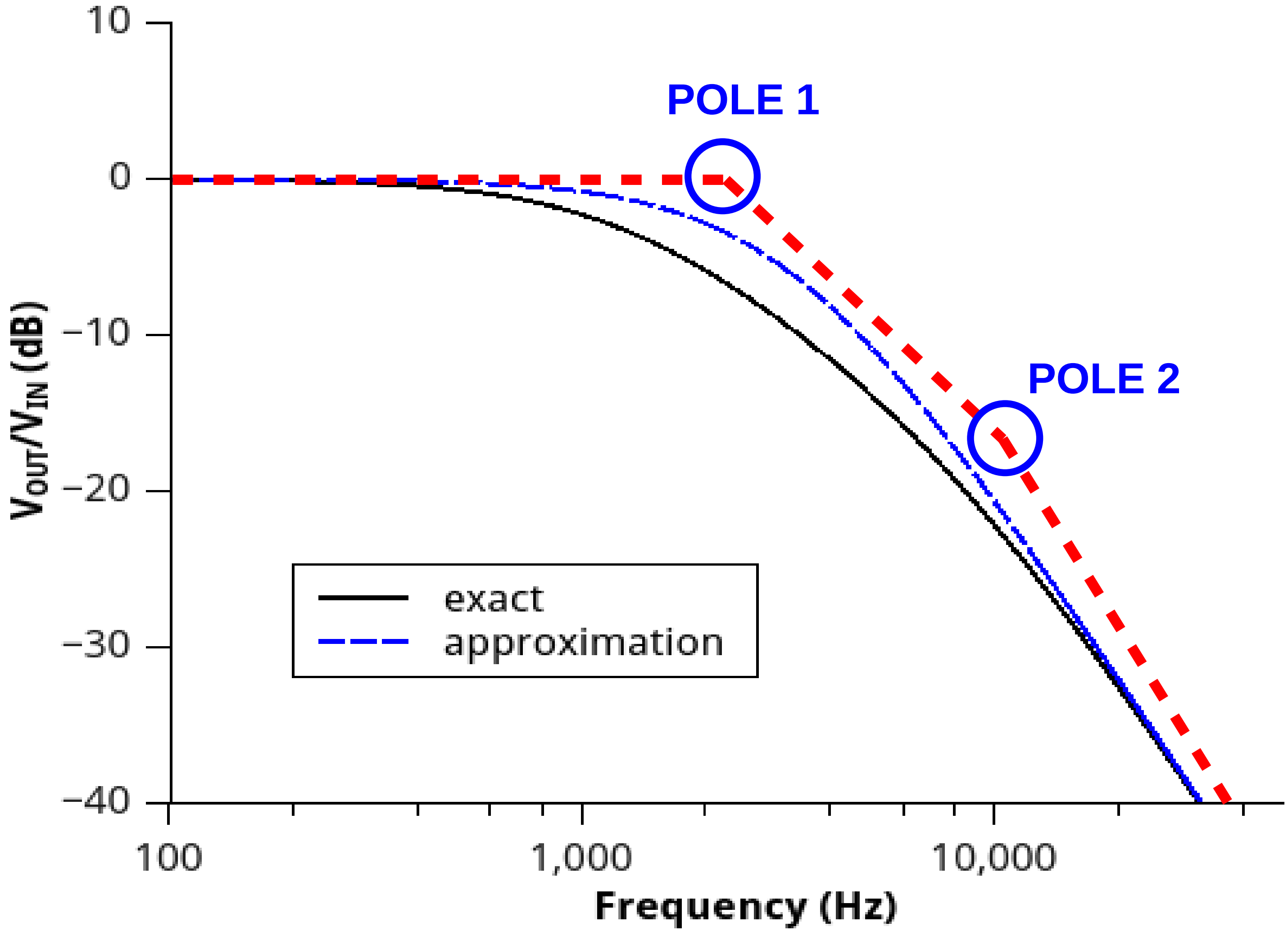
$C1$ = 10 nF $C2$ = 3 nF

f = 3.2 kHz f = 10.6 kHz

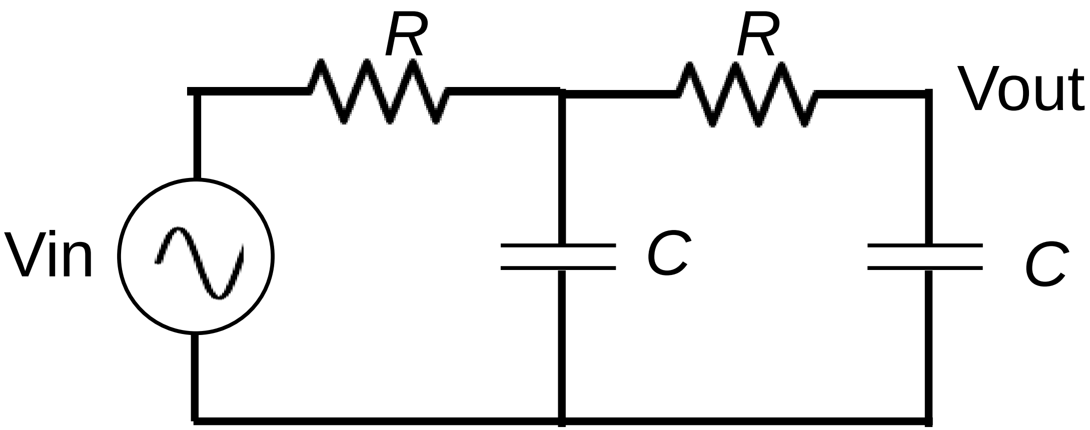




R1 = 5000 Ω *R2* = 500 Ω
C1 = 10 nF *C2* = 3 nF
f = 3.2 kHz *f* = 10.6 kHz



Example 3: 2-pole low-pass filter

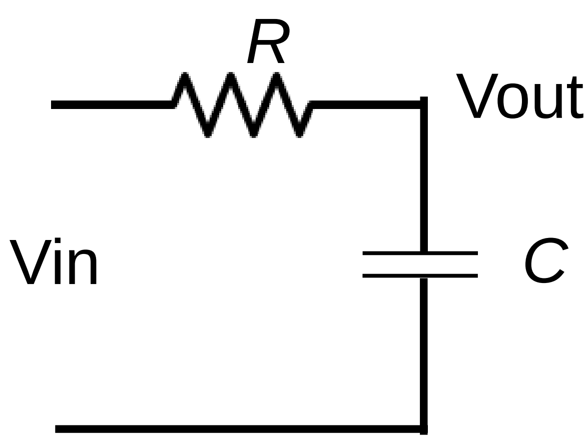


R = 5000 Ω
C = 10 nF
f = 3.2 kHz

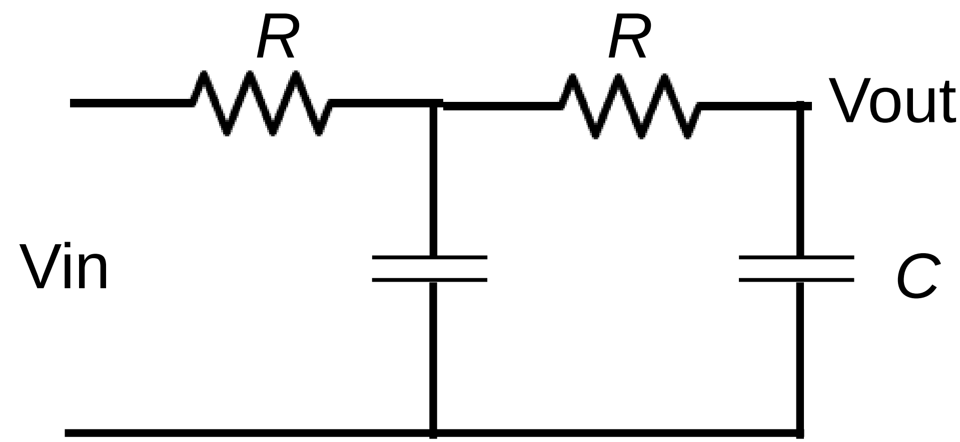
Assume "no loading"

$$\frac{V_{\mathrm{OUT}}}{V_{\mathrm{IN}}} \approx \underbrace{\left[\frac{1}{1 + j\omega RC}\right]}_{}^{2}$$

2 identical poles

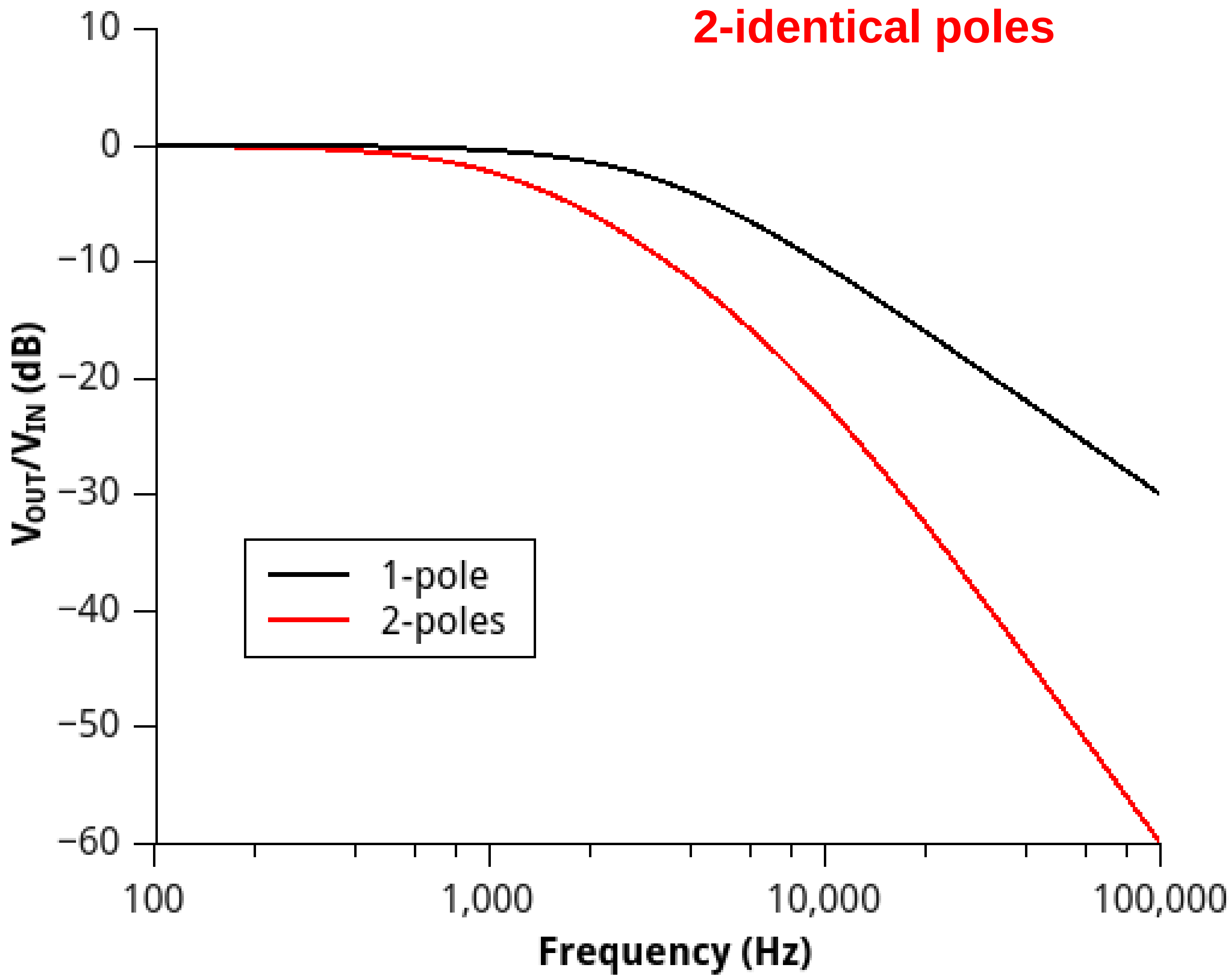


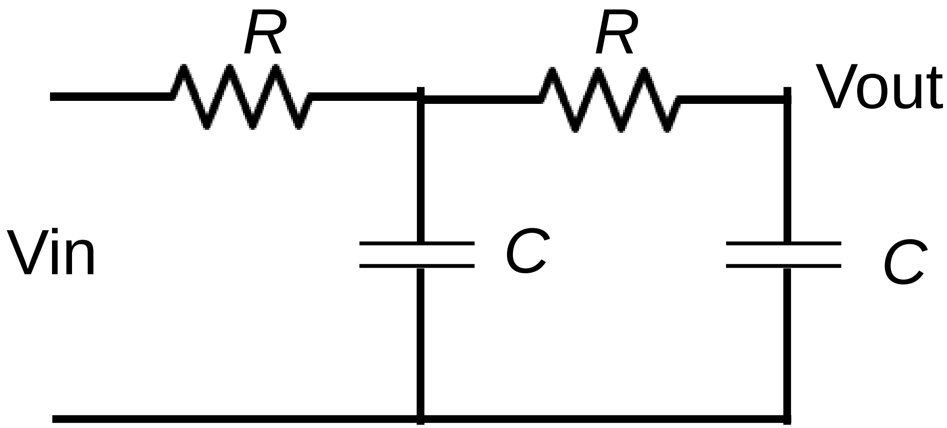
1-pole



2-identical poles

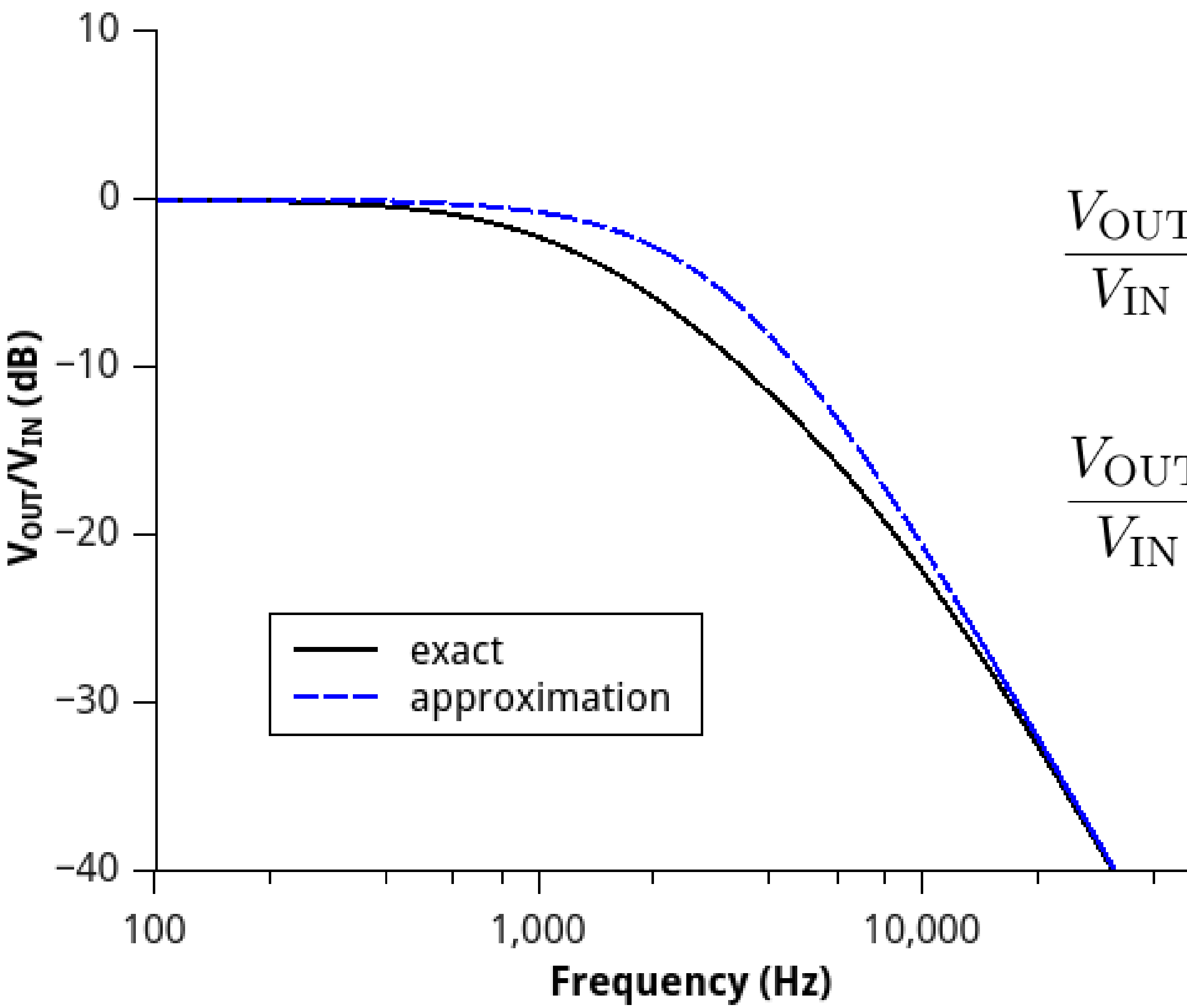
R = 5000 Ω
C = 10 nF
f = 3.2 kHz





2 identical poles

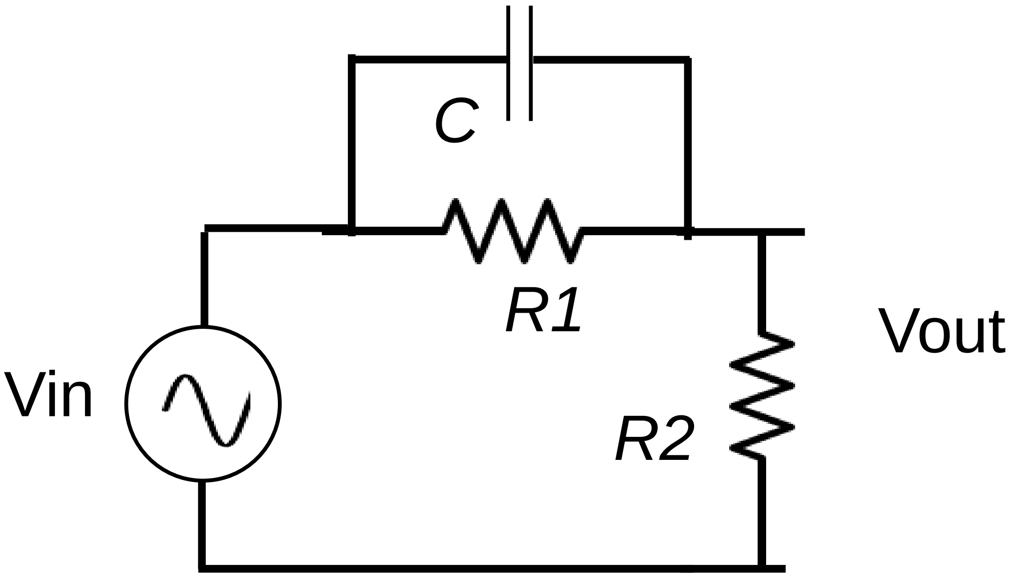
- Effect of Loading
- Analyze as uncoupled stages



$$\frac{V_{\mathrm{OUT}}}{V_{\mathrm{IN}}} = \frac{1}{1-(\omega RC)^2 + j\omega 3RC}$$

$$\frac{V_{\mathrm{OUT}}}{V_{\mathrm{IN}}} \approx \left[\frac{1}{1+j\omega RC}\right]^2$$

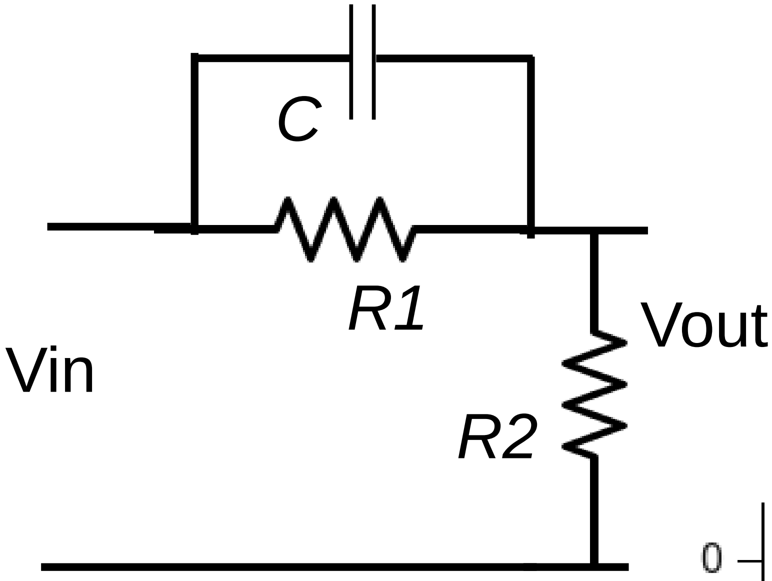
Example 4: 1-pole + 1-zero



ZERO in numerator

$$\frac{V_{\mathrm{OUT}}}{V_{\mathrm{IN}}} = \left[\frac{R2}{R1+R2}\right]\left[\frac{1+j\omega R1C}{1+j\omega C\,\frac{R1R2}{R1+R2}}\right]$$

POLE in denominator

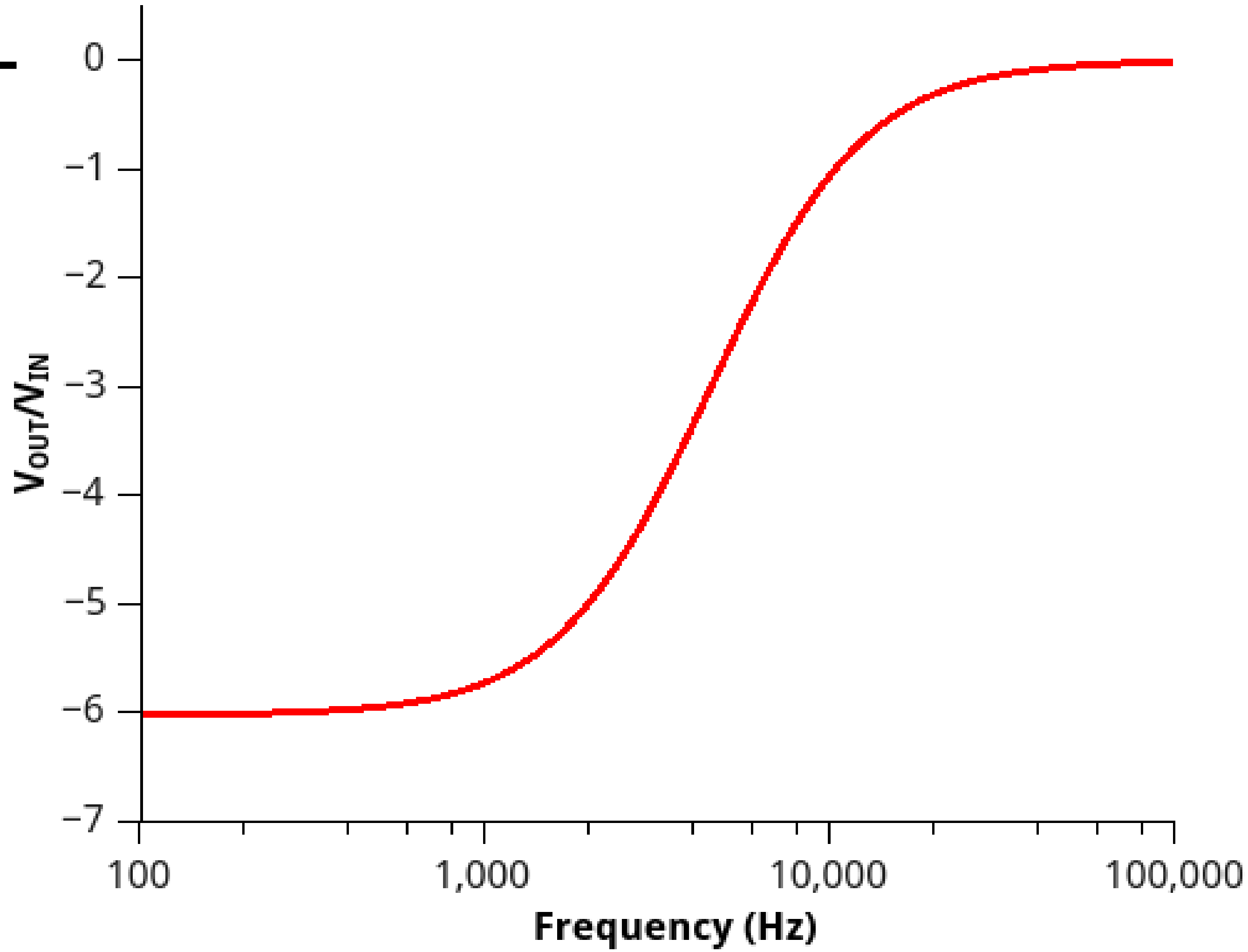


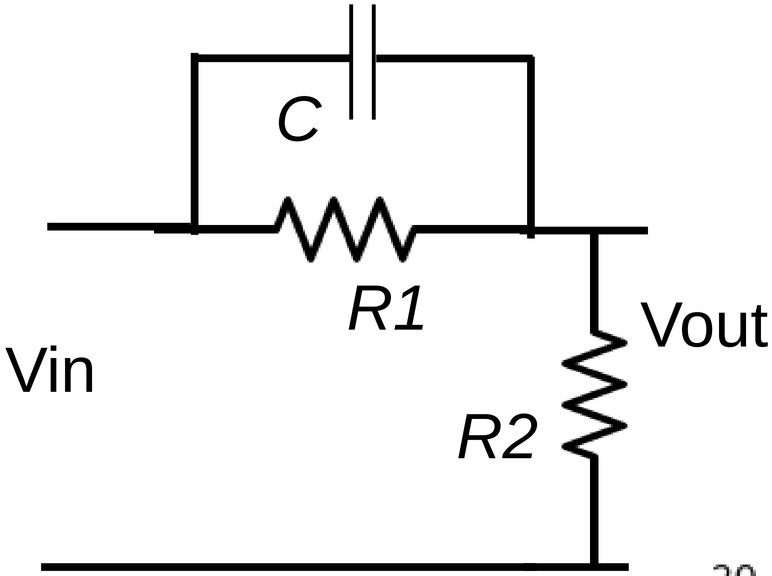
R1=*R2* = 500 Ω

C = 100 nF

f_{ZERO} = 3.2 kHz

f_{POLE} = 6.4 kHz





R1=*R2* = 500 Ω
C = 100 nF
f_{ZERO} = 3.2 kHz
f_{POLE} = 6.4 kHz