Chapter 10: Special Relativity

Time dilation
Length contraction
Energy, (relativistic) mass vs. rest mass
$E = m c^2$

Any questions or concerns so far?
Examples (such as cosmic ray muons) clear enough?

Quiz # 82: Velma passes Mort at high speed. His clock, as observed by her, runs at half of its normal speed. What is the value of the relativistic γ factor?

(a) ½ (b) 2 (c) 1 (d) −2 (e) not enough info given

Quiz # 83: A desperado riding on top of a train (v = 40 m/s or 0.04 km/s) fires a laser gun pointed forward. What is his gun's "muzzle velocity"?

(a) Can't answer w/o knowing more about the gun.
(b) c, i.e. 300,000 km/s
(c) 300,000.04 km/s
(d) 299,999.96 km/s
(e) Can't answer w/o knowing in which reference frame.

Quiz # 84: How fast does the tip of the laser beam move relative to the sheriff, who is standing on the ground beside the train? Pick from the same answers as in # 83 above.

(What would be the result according to Galilean relativity?)

Fascinating: time dilation presumably applies to biological clocks as well, with interesting consequences……”twin paradox” (10.6 in book, “Time Travel” → tested &*verified*w. atomic clocks)

But, come on, how can the traveling twin return “younger”? Wouldn’t he/she argue that the stay-at-home twin is younger? It’s “all relative”, after all.

Not quite….the two are *not* equivalent – why not? **Acceleration!**

Length Contraction: (<u>only</u> in direction of motion!) moving meter stick appears <u>shorter</u>, and by the same γ factor. But not narrower!

Without derivation, but the main point is easy to understand: in order to measure a length <u>properly</u>, the two end points must be measured…..

….*<u>simultaneously</u>* ! (unless object & meter stick at relative rest)

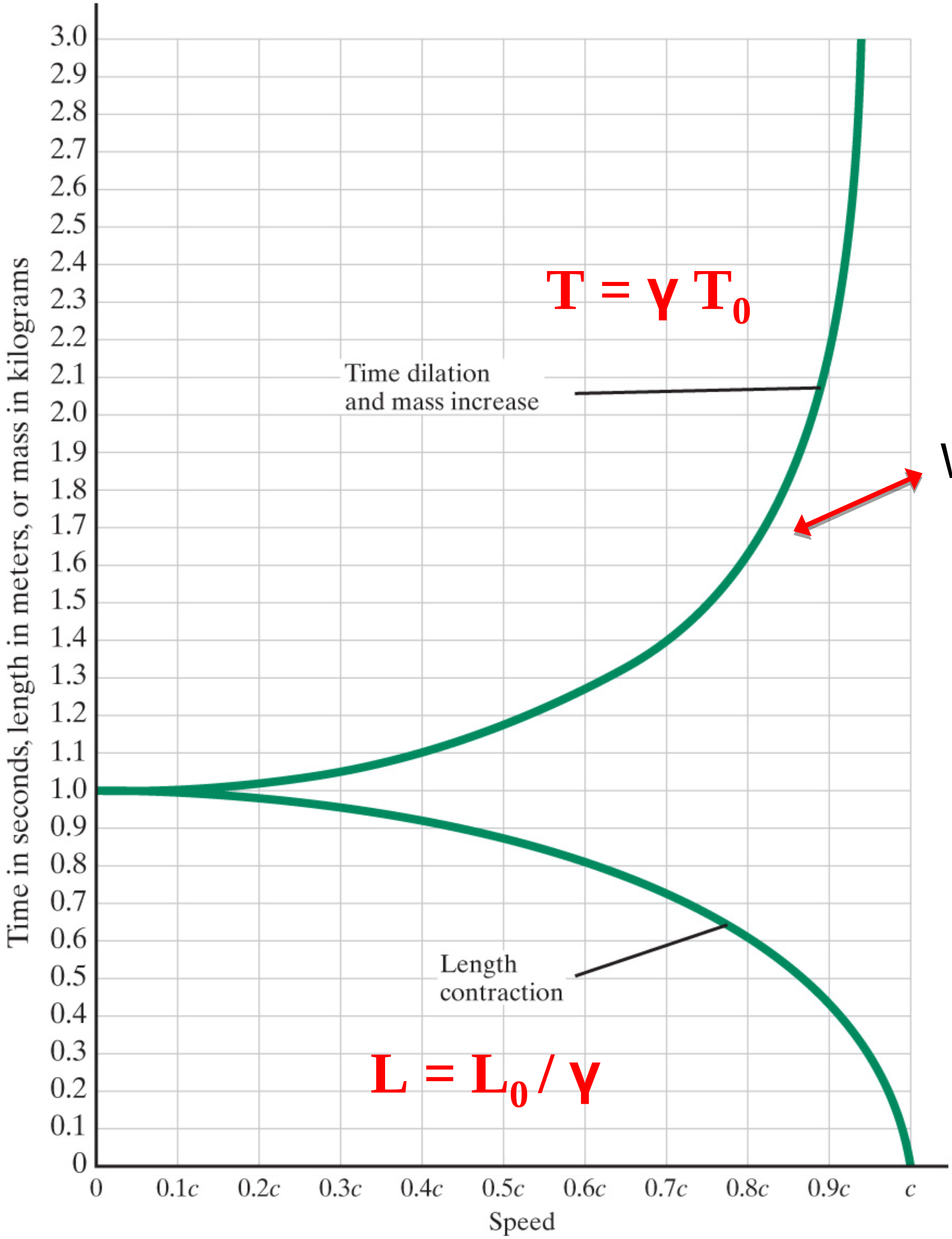
Important consequences: time intervals are relative (to the motion of the observer) → <u>distances are also relative</u> → <u>time & space are intertwined</u> or tangled up with each other!

Bottom line: $\mathbf{L = L_0 / \gamma}$ (L_0 = "proper" length, measured in its rest frame)

Remember the cosmic ray muon coming down? It sees the atmosphere shrunk by exactly the same factor (γ) as we see its lifetime dilated/extended – perfectly consistent measurements and results! (Don't forget: $\mathbf{T = \gamma\, T_0}$)

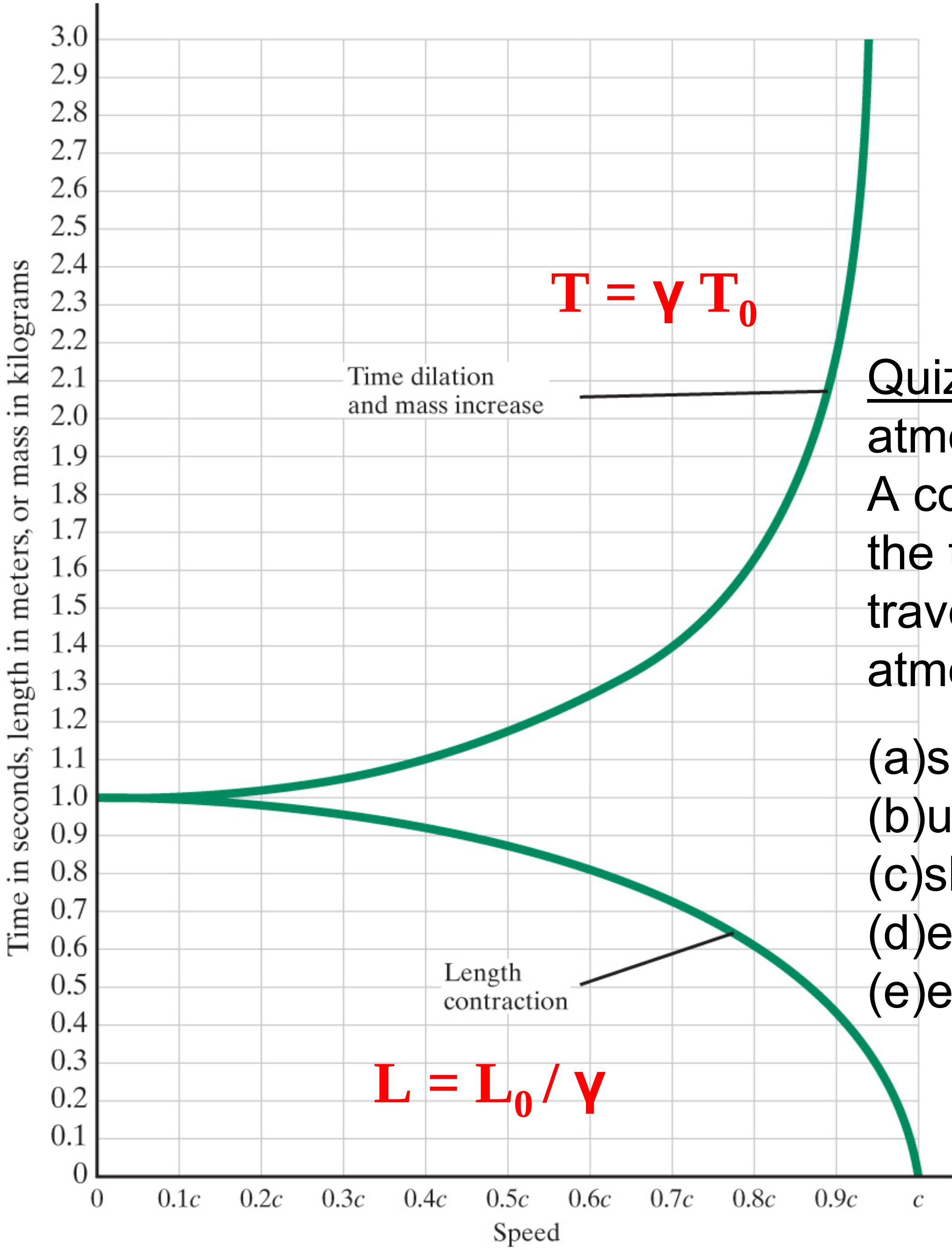
→ "Appearance of high speed objects"

The relativistic γ factor.



What's this "mass increase"?
(next topic…..)

$1 / \gamma$



<u>The relativistic γ factor</u>.

<u>Quiz # 85</u>: We measure Earth's atmosphere to be ~30 km thick. A cosmic ray muon, created at the top of the atmosphere and traveling at ~86% of c, sees the atmosphere

(a)shrunk to ~15 km.
(b)unchanged at ~30 km.
(c)shrunk to ~0.86 x 30 km.
(d)expanded ~30/0.86 km.
(e)expanded to ~60 km.

<u>$1 / \gamma$</u>

Einstein discovered that <u>*Newtonian mechanics needs modifications*</u> in order to be compatible with Special Relativity (to be "invariant"). This is in contrast to Maxwell's E & M, which he found to already be "invariant"!

Actually not too surprising….think a = F/m, and apply a constant F…..what would eventually happen to v ?

Relativistic prediction: a will become less & less because m (<u>*amount of inertia*</u>, remember?) will increase!
Consistent with the fact that a measures … what quantity?

a(cceleration) → distance / $(\text{time})^2$, and recall what happens to distance & time at relativistic speeds…….<u>*both drive a down*</u>.

Now the most important part:
So <u>*mass is relative*</u>(actually increases with γ, surprise!), BUT what about the amount of stuff, i.e. the number of atoms?
That's of course unchanged →<u>*rest mass*</u> m_0 → $m = \gamma\, m_0$

Let's clarify, since mass has now adopted a new meaning, i.e. not just the "amount of matter":

→At rest: $m = m_0$, i.e. the amount of matter (# of atoms) and also the amount of inertia.

→But: at speed an object's mass/inertia increases, while still containing the same # of atoms – crucial distinction!

Important:
So, a moving object has more inertia/mass, and therefore becomes harder & harder to accelerate….in fact, *a becomes infinitely small as the object approaches c, and m approaches infinity* (→ graph).

And that's why you can't accelerate an object right up to c!

Does anything move with v = c ? Yes → light/photons("γ") always travel with v = c! So photons must have $m_0 = 0$, and indeed they do.

Experimental proof of the increasing mass/inertia: subatomic particle accelerators find it harder & harder to bend the trajectory of charged particles in magnetic fields.

Very important bottom line distinction:

Objects with $m_0 = 0$ are stuck at $v = c$, whereas objects with $m_0 > 0$ are always stuck with $v < c$.

Examples:

$m_0 = 0$: photons/light, until some years ago neutrinos, gravitons (?)

$m_0 > 0$: all typical "matter" particles, i.e. electrons, protons, neutrons, and many more…..

C.E. 44: Oriented perpendicular to direction of v, and $v \approx 0.87$

C.E. 38: No, length contraction implies relative motion, which implies time dilation.

Quiz # 86: Mort's pool is 20 m long and 10 m wide. Velma flies lengthwise over it at 87% of c ($\gamma = 2$). How long and how wide will she observe it to be?

(a) 20 m long & 10 m wide.
(b) 10 m long & 20 m wide.
(c) 20 m long & 20 m wide.
(d) 40 m long & 10 m wide.
(e) 10 m long & 10 m wide.

Quiz # 87: Velma's spaceship has $m_0 = 10^4$ kg and $L_0 = 100$ m. She moves past Mort at a v such that $\gamma = 4$. She's in a hurry! Mort measures mass & length of her spaceship as

(a) 2500 kg & 100 m.
(b) 5000 kg & 25 m.
(c) 2×10^4 kg & 25 m.
(d) 4×10^4 kg & 50 m.
(e) none of the above.