Solutions to Exam #2, 11/25

1. Need to determine a first: \( \int_{0}^{0.55} x^2 dx = 1 \)
   
   a) \( \int (\frac{1}{3} x^2) dx = \frac{1}{3} x^3 \bigg|_{0}^{0.55} = 0.075 \)
   
   \( \Rightarrow a = \frac{1}{3} \)

   b) \( <x> = \int x (\frac{1}{3} x) dx = \frac{3}{4} \int x dx = \frac{3}{4} \) makes sense that \( <x> \) is greater than \( \frac{1}{2} \) since \( y(x) \) increases linearly.

2. \( H Y = E Y \)
   
   \( <E> = \int y^* H Y dx = \int y^* E Y dx = E \int y^* Y dx = E \) (of course!)

   \( E^2 = \int y^* H^2 Y dx = \int y^* H Y H Y dx = \int y^* H E Y dx = E \int y^* Y dx = E^2 \)

   \( \sigma_E = \sqrt{<E^2> - <E>^2} = \sqrt{E^2 - E^2} = 0 \) - surprising, of course since the system is in an eigenstate of \( H \)!

3. Energy splitting is proportional to \( \mu \) and to \( g_s \).
   
   \( \mu \) is proportional to \( \frac{1}{m} \), therefore \( \mu = \frac{0.511}{938.3} \mu_e = 5.45 \times 10^{-4} \mu_e \)

   Total \( E \) splitting ratio: \( 5.45 \times 10^{-4} \times 2.8 = 7.62 \times 10^{-4} \)

4. \( \mu^- \) would go into the 1s state, i.e. \( n = 1, l = 0 \), i.e. same state as the two \( e^- \) already there. No Pauli restrictions because the \( \mu^- \) is not identical to \( e^- \).
   
   Since \( a \), \( \times \frac{1}{m} \) the \( \mu^- \) would be a lot closer to the nucleus.
(4) continued:
Nuclear spin = 0, both $\mu$ and the $\pi$ have $l = 0$, so total of the two $\pi$ has to be 0 (remember!?). Leaves only $\mu = \frac{1}{2}$ as non-zero contribution.

Adding all these up gives $I_{\text{total}} = \frac{1}{2}$

(5) With spin-orbit: $l = 2$ splits into $j = \frac{3}{2}$ and $\frac{5}{2}$. And with $\Delta m_j = m_i - m_f = +1$ or $+2$ only the following transitions are allowed:

\[
\begin{array}{cccc}
\text{d}_{3/2} & m_f = +3/2 & & d_{5/2} \\
 & +1/2 & & +3/2 \\
 & -1/2 & & +1/2 \\
 & -3/2 & & -3/2 \\
 & -5/2 & & -5/2 \\
\end{array}
\]

same $\Delta E$

(no spin-orbit splitting!)

(2) i.e. $j = \frac{5}{2}$

Only 2 different $\Delta E$ (for transitions from $m_j = m_s = \pm \frac{1}{2}$) in addition you could have $\frac{1}{2} \rightarrow -\frac{1}{2}$ and $\frac{3}{2} \rightarrow -\frac{1}{2}$ transitions which would satisfy $\Delta m_j = +1$ or $+2$, but they have the same $\Delta E$ values.