PHYC 521: Graduate Quantum Mechanics I

Fall 2009

Homework Assignment #5

(Due October 19)

1- Exercise 9.4.2, Shankar, 2nd edition, page 244.

2- Exercise 8.6.2, Shankar, 2nd edition, page 234.

3- The imaginary time method is a powerful technique with many applications in quantum mechanics and quantum field theory. The goal of this problem is to demonstrate this by going through one such application.

Consider a system that has discrete energy spectrum. The energy of the ground state $|\psi_0\rangle$ is E_0.

(a) Write the matrix elements of the propagator in the position basis $U(x,t;x') \equiv \langle x|U(t)|x'\rangle$ in terms of the energy eigenstates $|\psi_n\rangle$ and energy eigenvalues E_n.

(b) Go to the imaginary time by making the transformation $\tau = it$, and write $U(x,t;x')$ in terms of τ. Shift the energy spectrum by adding a constant such that $E_0 = 0$. This amounts to multiplying $U(x,\tau;x')$ by an appropriate factor.

(c) In the limit that $\tau \to \infty$, show that $U(x,\tau;x) = |\psi_0(x)|^2$.

(d) Explicitly show the above for a harmonic oscillator. Use Eqs.(7.3.22) and (7.3.28) in Shankar for $U(x,t;x')$ and $\psi_0(x)$ in this case.