PHYC 521: Graduate Quantum Mechanics I

Fall 2009

Homework Assignment #2

(Due September 14)

1- Exercise 4.2.1, Shankar, 2nd edition, page 129.

2- Consider a physical system whose Hilbert space, which is three dimensional, is spanned by orthonormal basis consisting of three kets $|u_1\rangle, |u_2\rangle, |u_3\rangle$. In this basis the Hamiltonian of the system $\mathbf{H}$ and the two observabbles $\mathbf{A}$ and $\mathbf{B}$ have representations:

$$\mathbf{H} = \hbar\omega_\mathbf{0} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad \mathbf{A} = \mathbf{a} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad \mathbf{B} = \mathbf{b} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $\omega_\mathbf{0}$, $\mathbf{a}$, and $\mathbf{b}$ are real and positive constants. The state of the system at $t = 0$ is

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}|u_1\rangle + \frac{1}{2}|u_2\rangle + \frac{1}{2}|u_3\rangle\,.$$

(a) The energy of the system is measured at $t = 0$. What values can be found and with what probabilities? What is the mean value and uncertainty in energy?

(b) If one measures observable $\mathbf{A}$ at $t = 0$ what values can be found and with what probabilities? What is the state vector immediately after the measurement?

(c) Find $|\psi(t)\rangle$.

(d) Calculate $\langle\mathbf{A}\rangle(t)$ and $\langle\mathbf{B}\rangle(t)$ for $t > 0$. What comments can be made?

(e) What results can be obtained if observable $\mathbf{A}$ is measured at time t. Repeat for B. Interpret.

3- Show that

(a) For a wavefunction $\psi(x) = c\psi_r(x)$, where ψ_r is real and c is an arbitrary (real or complex) constant, the mean value of momentum $\langle P \rangle = 0$.

(b) For a wavefunction $\psi(x)$ $(-\infty < x < \infty)$, where $\psi(-x) = e^{i\alpha}\psi(x)$ (α is a real constant), the mean value of position $\langle X \rangle = 0$.