2- Consider a physical system whose Hilbert space, which is three dimensional, is spanned by orthonormal basis consisting of three kets $|u_1\rangle, |u_2\rangle, |u_3\rangle$. In this basis the Hamiltonian of the system $H$ and the two observables $A$ and $B$ have representations:

$$
H = \hbar \omega_0 \begin{bmatrix}
1 & 0 & 0 \\
0 & 2 & 0 \\
0 & 0 & 2 \\
\end{bmatrix}
$$

$$
A = a \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0 \\
\end{bmatrix}
$$

$$
B = b \begin{bmatrix}
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
$$

where $\omega_0$, $a$, and $b$ are real and positive constants. The state of the system at $t = 0$ is

$$
|\psi(0)\rangle = \frac{1}{\sqrt{2}} |u_1\rangle + \frac{1}{2} |u_2\rangle + \frac{1}{2} |u_3\rangle.
$$

(a) The energy of the system is measured at $t = 0$. What values can be found and with what probabilities? What is the mean value and uncertainty in energy?

(b) If one measures observable $A$ at $t = 0$ what values can be found and with what probabilities? What is the state vector immediately after the measurement?

(c) Find $|\psi(t)\rangle$.

(d) Calculate $\langle A \rangle(t)$ and $\langle B \rangle(t)$ for $t > 0$. What comments can be made?

(e) What results can be obtained if observable $A$ is measured at time $t$. Repeat for $B$. Interpret.
3- Show that

(a) For a wavefunction \( \psi(x) = c\psi_r(x) \), where \( \psi_r \) is real and \( c \) is an arbitrary (real or complex) constant, the mean value of momentum \( \langle P \rangle = 0 \).

(b) For a wavefunction \( \psi(x) \) \((-\infty < x < \infty)\), where \( \psi(-x) = e^{i\alpha}\psi(x) \) (\( \alpha \) is a real constant), the mean value of position \( \langle X \rangle = 0 \).