PHYC 521: Graduate Quantum Mechanics I

Fall 2009

Homework Assignment #10

(Due December 7)

1-Exercise 12.5.3, Shankar, 2nd edition, page 329.

2-Exercise 12.6.1, Shankar, 2nd edition, page 340.

3-The aim of this problem is to demonstrate the addition of angular momentum by working out an explicit example. Consider three spin-1/2 particles. The spin-up and spin-down states for each particle are denoted by $|+\rangle$ and $|-\rangle$ respectively. The angular momentum operator in the three-particle Hilbert space $\mathcal{H}_{1\otimes 2\otimes 3}$ is defined as

$$\vec{J} = \vec{J_1} \otimes I_2 \otimes I_3 + I_1 \otimes \vec{J_2} \otimes I_3 + I_1 \otimes I_2 \otimes \vec{J_3}\,,$$

where the subscripts $1, 2, 3$ refer to the single-particle Hilbert spaces $\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3$ respectively.

(a) Consider the state $|++{+}\rangle$ in $\mathcal{H}_{1\otimes 2\otimes 3}$. Show that this is a simultaneous eigenstate of $|\vec{J}|^2$ and J_z. Find the corresponding eigenvalues.

(b) Use the lowering operator J_- to find all states with the same total angular momentum. Determine the eigenvalue of each state under J_z.

(c) These states are symmetric under the exchange of particles $1 \leftrightarrow 2, 2 \leftrightarrow 3, 1 \leftrightarrow 3$ and span a four-dimensonal subspace of $\mathcal{H}_{1\otimes 2\otimes 3}$. As we saw before, the other four independet states in $\mathcal{H}_{1\otimes 2\otimes 3}$ are mixed. What is the total angular monentum of those states? (Hint: Argue what values of j are allowed based on the eigenvalues of mixed states under J_z.)