PHYC 521: Graduate Quantum Mechanics I

Fall 2009

Final Exam

12/16/2009, 10:00 am-12:00 pm

Instructions:

- This is an open-book open-note exam, all reference material allowed.
- Two problems, equally weighted.

1- The Hamiltonian operator for a harmonic oscillator in two dimesnions is given by

$$\mathbf{H} = \frac{\mathbf{P_x^2}}{2m} + \frac{\mathbf{P_y^2}}{2m} + \frac{3}{2}m\omega^2\mathbf{X^2} + \frac{3}{2}m\omega^2\mathbf{Y^2} - m\omega^2\mathbf{XY}\,.$$

(a) Find the energy eigenvalues for this oscillator.

(b) Is there any degeneracy in the energy eigenstates? Comment.

(c) Find the three lowest-lying energy eigenvalues. How do the corresponding eigenstates transform under parity transformation?

2- Consider a system consisting of two free and non-interacting *identical* spin-1/2 fermions in three dimensions. The system is in a state with energy E and the orbital part of the wavefunction is given by

$$\Psi = \exp(i\vec{k}.\vec{R})\psi(\vec{r}) \,,$$

where $\vec{R} = (\vec{r}_1 + \vec{r}_2)/2$ and $\vec{r} = \vec{r}_1 - \vec{r}_2$ are the center-of-mass and relative coordinates respectively. Assume that ψ has the following functional form in the spherical coordinates

$$\psi(\vec{r}) = f(r, \sin\theta)e^{2i\phi} \,.$$

(a) How does Ψ transform under the exchange of two particles $1 \leftrightarrow 2$? Use this and write down the spin part of the wavefunction.

(b) Show that ψ is an eigenstate of L_z and find the corresponding eigenvalue.

(c) ψ can be expanded in terms of energy eigenstates

$$\psi = \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} a_{lm} R_{El}(r) Y_{lm}(\theta, \phi) \,.$$

Based on parts (a),(b) what can you say about the coefficients a_{lm}?

(d) Use the result of part (c) to determine f at $r = 0$ and also find a lower bound on $\langle L^2 \rangle$.