Lecture 23 04/22/2019

Scattering (Cont'd)

Let us consider two important examples of scattering.

Rayleigh Scattering

In this case the scatterer is a small dielectric object (like an atom) that has a polarizability γ such that:

$$\vec{P} = \varepsilon_0 \gamma \vec{E}_{inc}$$

For example, for a dielectric sphere that we discussed, we have:

$$\gamma = 4\pi a^3 \left(\frac{\varepsilon_r - 1}{\varepsilon_r + 2} \right)$$

The total scattering cross section for Rayleigh scattering, in terms of γ, then is:

$$\sigma_R = \frac{8\pi}{3} k^4 \left| \frac{\gamma}{4\pi} \right|^2 = \boxed{\frac{k^4}{6\pi} |\gamma|^2}$$

For a bound electron with natural frequency ω_0, at frequencies far from ω_0, we showed that:

$$\gamma \approx \frac{e^2}{\varepsilon_0 m_e (\omega^2 - \omega_0^2)}$$

If the number density of bound electrons is n_e, we then have:

$$\gamma \approx \frac{n_e e^2}{\varepsilon_0 m_e (\omega^2 - \omega_0^2)}$$

For $\omega \ll \omega_0$, this results in:

$$\sigma_R = \frac{k^4}{6\pi} \frac{n_e^2 e^4}{\varepsilon_0^2 m_e^2 \omega_0^4}$$

We see that in this case $\sigma_{tot} \propto \omega^4$, which is the reason why the sky is blue.

Thomson Scattering

This is the scattering of an electromagnetic wave by a free electron. In this case, we can set $\omega_0 = 0$ in above, and hence:

$$\gamma = \frac{e^2}{\varepsilon_0 m_e \omega^2}$$

This results in:

$$\sigma_T = \frac{k^4}{6\pi} \frac{e^4}{\varepsilon_0^2 m_e^2 \omega^4} = \frac{c^4 e^4}{6\pi \varepsilon_0^2 m_e^2}$$

The Thomson scattering cross section can be expressed in terms of the "classical electron radius" $r_e = \frac{e^2}{4\pi\varepsilon_0 m_e c^2}$ as follows:

$$\boxed{\sigma_T = \frac{8\pi}{3} r_e^2 \simeq \frac{2}{3} \times 10^{-24}\ cm^2}$$

This classical derivation is a good approximation of the cross section for scattering of a photon off a free electron, called "Compton scattering", that can be calculated in quantum electrodynamics when $\omega \ll \frac{m_e c^2}{\hbar}$.

Finally, let us discuss extinction of the forward wave because of scattering (and possibly absorption). Consider a plane wave that is propagating through a dilute gas of scatterers. This wave undergoes scattering (and possibly absorption), and hence attenuated in the forward direction. If the number density of scatterers is n and the total cross section (for scattering +

plus absorption) is σ_{tot}, we will have (I being the intensity):

$$\frac{dI}{dz} = -n\sigma_{tot} I \Rightarrow I(z) = I(0)\, e^{-n\sigma_{tot} z}$$

We can define the extinction coefficient $\alpha \equiv n\sigma_{tot}$, which is related to the total scattering cross section at all angles (plus absorption). This is basically what the "optical theorem" is about.

We note that in a dense medium there can be multiple scatterings, which must be included.