Physics 511: Electrodynamics

Spring 2019

Midterm Exam #1

March 6, 2019

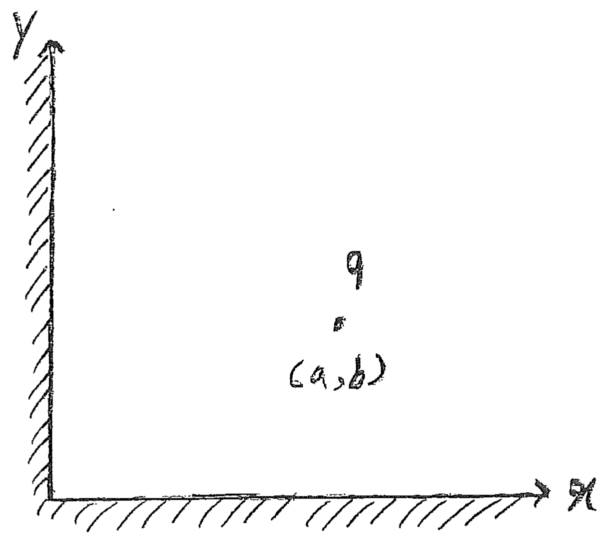
Instructions:

- Do any 1 of problems 1, 2 and any 1 of problems 3, 4. All problems carry equal weight.
- This is an open-book open-note exam.

1- The three-dimensional region $x \geq 0$, $y \geq 0$ is bounded by conducting planes held at zero potential. A point charge q is placed at the point $(a, b, 0)$ in this region.

(a) Find the surface charge density on the conductors.

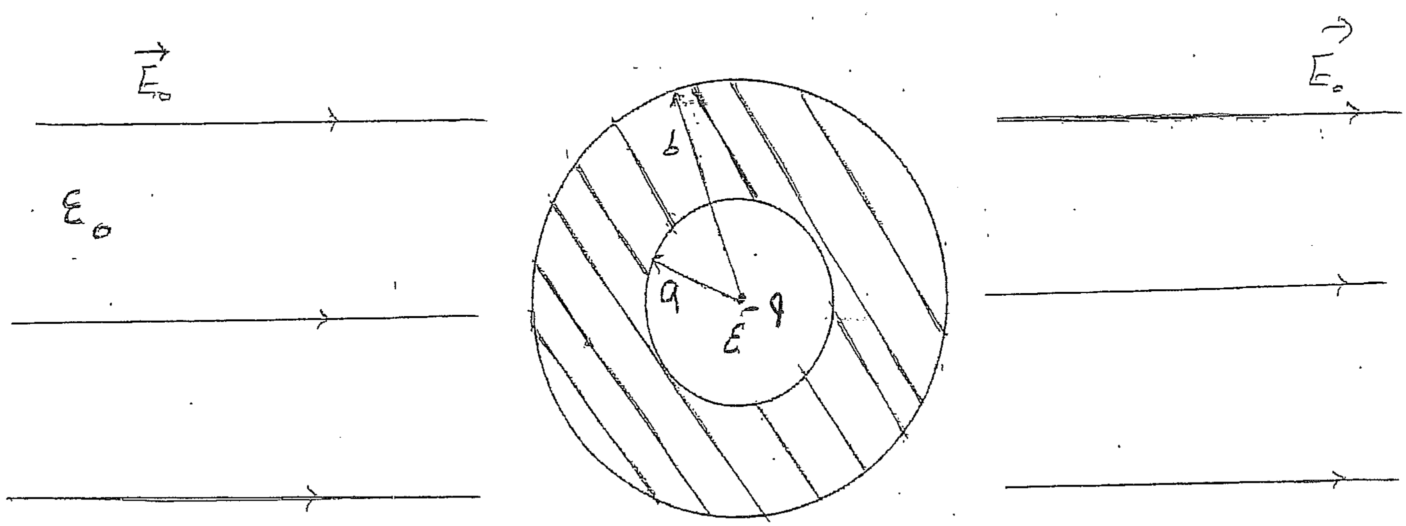
(b) Determine the magnitude and direction of the force acting on the charge q. What is the amount of work done to bring the charge q from infinity to the point $(a, b, 0)$.



2- A conducting spherical shell of inner and outer radii a and b is placed in a uniform external electric field $\vec{E}_0$. The space at $r < a$ is filled with dielectric material of permittivity ϵ, and a point charge $-q$ is located at its center.

(a) Determine the electric potential in all regions.

(b) Find the surface density of free and bound charges on the interfaces at at $r = a$ and $r = b$.

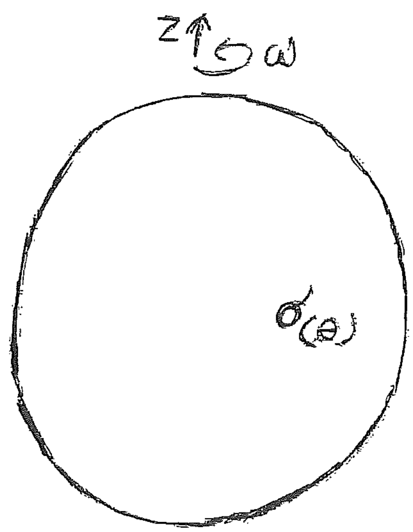


3- Consider a sphere of radius R rotating at angular velocity ω about the z axis, with a surface charge density given by $\sigma(\theta) = \sigma_0 \cos\theta$. Show that the vector potential is given by:

$$\vec{A}(\vec{x}) = \frac{4\pi}{5} \frac{\sigma_0 \omega}{c} r^2 \cos\theta \sin\theta \hat{\phi} \qquad r \leq R, \tag{1}$$

$$\vec{A}(\vec{x}) = \frac{4\pi}{5} \frac{\sigma_0 \omega}{c} \frac{R^5}{r^3} \cos\theta \sin\theta \hat{\phi} \qquad r > R. \tag{2}$$

What are the non-vanishing magnetic multipole moments for this system?



4- A circular loop of radius a carrying a current I_a is placed parallel to another loop of radius b carrying a current I_b. The loops are co-axial but the smaller loop is centerd at a height h above the larger one. The currents I_a and I_b flow in opposite directions. Consider the limiting case when $a \ll b,\ h$.

(a) What is the force expeienced by the smaller loop?

(b) How does the force change when the direction of the current in (1) one of the loops is reversed; (ii) both loops is reversed?

(Hint: In the limit $a \ll b,\ h$ the smaller loop may be considered as a magnetic dipole.)