PHYC 467: Methods of Theoretical Physics II

Spring 2013

Homework Assignment #9

(Due May 2, 2013)

1- Show that the bilinears $\psi_L^\dagger \psi_L$ and $\psi_R^\dagger \psi_R$ made out of left-handed and right-handed Weyl spinors ψ_L and ψ_R respectively are invariant under rotations but not under boosts. Also show that bilinears $\psi_R^\dagger \psi_L$ and $\psi_L^\dagger \psi_R$ are invariant under a general Lorentz transformation.

2- Use the relation $\sigma^2 \sigma^i \sigma^2 = -\sigma^{i*}$ for the Pauli matrices to show that

$$\sigma^2 \Lambda_L^{-1} \sigma^2 = \Lambda_L^T \,,$$

where Λ_L denotes a Lorentz transformations acting on a left-handed Weyl spinors ψ_L.

(a) Use this identity to prove that $\sigma^2 \psi_L^*$ behaves like a right-handed Weyl spinor under the Lorentz transformations.

(b) Show that the bilinear $\psi_L^T \sigma^2 \psi_L$ is invariant under a general Lorentz transformation.

(The four-component spinor Ψ_M that is constructed from the direct sum of ψ_L and $\psi_R' = \sigma^2 \psi_L^*$ is called a Majorana spinor.)

3- As we saw in the class, the Poincare group has two Casimir operators. What is the rank of the Poincare group? Discuss the consistency between the number of Casimir operators and the group rank and Racah's theorem in this case.

Hint: Use the commutation relations among the generators of the Poincare group to find whether it is simple, semisimple, etc.