PHYC 467: Methods of Theoretical Physics II

Spring 2013

Homework Assignment #8

(Due April 25, 2013)

1- As we discussed before, the direct product of two $SU(2)$ representation with angular momenta j_1 and j_2 is decomposed into a direct sum of irreducible representations in the following way:

$$\mathbf{j_1} \otimes \mathbf{j_2} = (\mathbf{j_1} + \mathbf{j_2}) \oplus (\mathbf{j_1} + \mathbf{j_2} - 1) \oplus ... \oplus (|\mathbf{j_1} - \mathbf{j_2}|) \,.$$

Verify this relation by using the standard Young diagrams for $SU(2)$.

2- Use the Young diagrams to find how **6** and **8** representations of $SU(3)$ are decomposed into irreducible representations of its $SU(2)$ subgroup(s). Verify your result by using the weight diagrams for these representations.

3- In grand unified theories (GUTs) the strong, weak, and electromagnetic interactions in the standard model that are described by the $SU(3)_C \otimes SU(2)_W \otimes U(1)_Y$ symmetry group are embedded into a single larger group. An example is the $SU(5)$ grand unified theory, where the elementary fermions (i.e., quarks, leptons and their antiparticles) of each generation belong to $\bar{\mathbf{5}} \oplus \mathbf{10}$ reducible representation of $SU(5)$.

The **10** representation contains three color states of the up quark, three color states of the down quark, three color states of the up anti-quark, and positron (which has no color). This implies that $\mathbf{10}_{SU(5)} = \mathbf{3}_{SU(3)} \oplus \mathbf{3}_{SU(3)} \oplus \bar{\mathbf{3}}_{SU(3)} \oplus \mathbf{1}_{SU(3)}$. Derive this decomposition by using the Young diagrams.

Hint: You may do the decomposition at two steps $SU(5) \to SU(4) \to SU(3)$.