PHYC 467: Methods of Theoretical Physics II

Spring 2013

Homework Assignment #6

(Due April 4, 2013)

1- Consider a unitary representation of a simple Lie group that is spanned by an orthonormal basis $|i\rangle$ ($1 \leq i \leq n$). Generators of the group in this representation are denoted by T_a. What do the group generators look like in the representation spanned by the orthonormal basis $\langle i|$ ($1 \leq i \leq n$)? Conclude that the two representations are complex conjugate of each other.

Hint: Compare the action of infinitesimal group elements in the two representations.

2- Prove that the fundamental representation of $SU(2)$ is real by showing that T_i and $-T_i^*$ ($1 \leq i \leq 3$) are related through $\sigma_2^\dagger T_i \sigma_2 = -T_i^*$. Use this to comment on whether an n-dimensional representation of $SU(2)$ (n being an arbitrary integer) is real or complex.

Note: To be more precise, the fundamental representation of $SU(2)$ is pseudo-real since the unitary matrix that relates T_i and $-T_i^*$ is antisymmetric in this case.

3- Consider the direct product of two **3**'s of $SU(3)$ and their decomposition: $\mathbf{3} \otimes \mathbf{3} = 6 \oplus \bar{\mathbf{3}}$. Construct all the states of **6** in terms of the states of **3**. Without explicit derivation, argue why the remaining three states in the direct product space form a $\bar{\mathbf{3}}$ instead of a **3**.