PHYC 467: Methods of Theoretical Physics II

Spring 2013

Homework Assignment #1

(Due February 5, 2013)

1- As we saw in the class, the n-th roots of unity form a cyclic group of order n under multiplication. Show that if m is a divisor of n, then the said group has a subgroup of order m. Discuss whether this subgroup is cyclic too.

2- Find the subgroups H, H' of the symmetric group S_4 that leave the polynomials $x_1x_2 + x_3 + x_4$ and $x_1x_2 + x_3x_4$ invariant respectively. Show that H' contains H as a subgroup.

3- Construct the symmetry group of an equilateral triangle (denoted by $C_{3\nu}$ in crystallography). Write down its multiplication table and show that it is isomorphic to the symmetric group S_3. Find the number of inequivalent irreducible representations of this group and their dimensionality.

4- If a set of matrices Γ is a representation of a group G, show that Γ^* (whose matrices are complex conjugates of the corresponding matrices of Γ) is also a representation of G. Show that the same is not true for Γ^{-1} (whose matrices are the inverse of the corresponding matrices of Γ) and $\Gamma^\dagger$ (whose matrices are hermitian conjugates of the corresponding matrices of Γ), unless G is an abelian group.

5- Let $\Gamma^{(i)}$ and $\Gamma^{(j)}$ be two inequivalent irreducible representations of of a group G. Show that the direct product representation $\Gamma^{(i)} \bigotimes \Gamma^{(j)*}$ does not contain the identity representation. Show also that the direct product of an irreducible representation with its own complex conjugate representation contains the identity representation once and only once.